

At-issue Proposals and Appositive Impositions in Discourse*

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Abstract

Potts (2005) and many subsequent works have argued that the semantic content of appositive (non-restrictive) relative clauses, e.g., the underlined material in *John, who nearly killed a woman with his car, visited her in the hospital*, must be in some way separate from the content of the rest of the sentence, i.e., from at-issue content. At the same time, there is mounting evidence from various anaphoric processes that the two kinds of content must be integrated into a single, incrementally evolving semantic representation. The challenge is how to reconcile this informational separation with these pervasive anaphoric connections. We propose a dynamic semantic account that accomplishes this by taking appositive and at-issue content to involve two different kinds of updates to the Context Set (CS). Treating the context set as a distinguished propositional variable, p^{CS} , we argue that appositives directly *impose* their content on the CS by eliminating possible values assigned to p^{CS} . In contrast, we treat at-issue assertions as introducing a new propositional dref and *proposing* that p^{CS} be updated with its content, subject to addressee's response. In addition to capturing the behavior of appositives in discourse, we show that the account can be extended to capture the projection of appositive content past various sentential operators.

Keywords: appositives, dynamic semantics, assertion, projection, context.

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1 Introduction

Since Potts (2005), it has been widely accepted that the content of appositive (non-restrictive) relative clauses¹ must be semantically separate from the at-issue content. While we develop a more specific conception of what it means to be (not) at-issue as the paper progresses, we can informally equate ‘at-issue’ with the ‘main point’ of an utterance. We take it that at-issue content excludes presuppositions and implicatures among other things. As an example of this intuitive separation, consider the minimal pair in (1-2) below. (1), which contains an appositive relative clause, seems to have a different interpretation than (2), where the same content is instead conjoined.

- (1) John, who played tennis with a woman, played golf with her too.
- (2) John played tennis with a woman and played golf with her too.

There are two, potentially related differences between appositive and at-issue content. First, the two kinds of content behave differently in discourse, e.g., with respect to their ability to resolve questions under discussion (we’ll return to this in due course). Second, appositive content fails to interact with at-issue sentential operators such as negation and modals, i.e., it is projective. Potts (2005) proposes to compositionally account for this latter separation by positing *multidimensional* semantic representations, i.e., semantic representations that are sets or tuples of whatever sort of denotations one assigns to (subparts of) simple at-issue assertions.

While there are clear differences between appositive and at-issue content, there are also similarities. For instance, the appositive boundary in (2) does not disrupt the anaphoric link between *her* and *a woman*. This raises a question about how to extend the notion of multidimensionality originally formalized in a static, exclusively truth-conditional semantic framework, to a dynamic framework in which the denotations of expressions guide the interpretation of anaphora. Potts (2005: 42) describes multidimensionality as “a formal implementation of the independence property”, which states that CIs “are logically and compositionally independent of what is *said* (in the favored sense)”. In the wake of the ‘dynamic turn’, it is now common to take (certain aspects

¹Since we are dealing almost exclusively with appositive relative clauses rather than nominal appositives, we will use the term ‘appositive’ and ‘appositive relative clause’ interchangeably. On the few occasions where we do address nominal appositives (e.g., §5.2), we will be explicit in using the term ‘nominal appositive’. Following the discussion in Potts (2005: 93-94), we will refrain from using the term ‘non-restrictive’.

of) both intra- and inter-sentential anaphora to be part of the recursive definition of truth and satisfaction, i.e., part of semantics proper. From a purely formal perspective then, it seems quite natural to expect that a multidimensional dynamic semantics would make the strong prediction that the at-issue and appositive dimensions will be wholly separate with respect to all such phenomena (see for example the discussion in [Nouwen 2007](#)).

It is not clear that this is in fact what [Potts \(2005\)](#) envisions, and indeed, he points out (pp. 51-54) cases of individual anaphora across ‘dimensions’ analogous to (1). Similar anaphoric facts have been discussed in more depth by [Nouwen \(2007\)](#) and [Potts \(2009\)](#), and [Amaral et al \(2007\)](#) discuss not only boundary-crossing anaphora, but also presupposition. While such observations have been noted, we find that the systematicity with which anaphoric processes of nearly all sorts can span the at-issue/not-at-issue divide has been underappreciated and that formal accounts have reflected this.

Drawing heavily on examples from [Davies \(2008-\)](#)’s Corpus of Contemporary American English (COCA), we show that this pattern holds of anaphora and presupposition quite generally, as well as most ellipsis processes. An adequate account of appositives, then, must capture the sense in which the appositive’s informational content is separate, but do so in a way that naturally allows for an understanding of the rich sorts of connections that are possible between the two contents.

One way to approach this problem is by providing a fundamentally *unidimensional* semantics in which the differential discourse and projective behavior of appositives is captured within a single, integrated meaning representation for the sentence/discourse. Alternatively, we could investigate how the *notion* of multidimensionality can be extended to incorporate systematic connections between various meaning dimensions. We tend to favor the former conception and we will frequently talk about the challenge summarized in the previous paragraph in those terms, though we acknowledge that both ways of thinking about the analysis we develop may prove fruitful and lead to a more nuanced view of the multidimensional vs. unidimensional distinction.

In this vein, we propose an analysis in which appositive and at-issue content are distinguished principally in *how* they enter the Common Ground (CG, used here in the sense of [Stalnaker 1978](#)). For the sake of simplicity, the remainder of the paper will generally talk directly in terms of the Context Set (CS) rather than the CG. We will follow [Farkas & Bruce \(2010\)](#) (and references therein) and will take at-issue meanings to be *proposals* to update the input CS (a feature of assertion that is already present in [Stalnaker 1978](#), but deemphasized). In contrast, appositive content is *imposed* on the CS and not up for negotiation by normal

means.

Formally, we implement this idea within a dynamic semantics system based on Dynamic Predicate Logic (DPL, [Groenendijk & Stokhof 1991](#)), treating appositive and at-issue content as constituting two different ways of *updating* the CS. We choose DPL because its syntax and semantics are most obviously related to the very familiar classical first-order logic syntax and semantics. This allows us to focus on the main features of our proposal rather than on orthogonal details of the general semantic framework.

Our DPL-based system enables us to provide a precise formulation of the discourse status of both appositive and at-issue content. Equally importantly, using a semantic framework that conceptualizes natural language interpretation as a process of *incremental* update enables us to account for phenomena like anaphora, ellipsis, and presupposition that systematically cross the appositive/at-issue meaning boundary (see [Amaral et al 2007](#) for more discussion of this point).

Accounting for this boundary-crossing behavior is often a necessary prerequisite if we want to determine the truth-conditional content itself of both the appositive clause and the remainder of the sentence/discourse. For example, the pronoun *her* in the main clause of (1) needs to find its antecedent within the appositive for the at-issue content to be fully determined. Similarly, the presupposition of *too* must be satisfied by the *content* of the appositive if we want to fully and correctly determine the at-issue content of that example.

The paper is structured as follows: §2 argues against a strongly multidimensional semantics by showing the robust boundary-crossing behavior of a wide variety of phenomena, including anaphora, presupposition and ellipsis. Section §3 presents the core of the account: a dynamic semantics that builds a unidimensional, incrementally-evolving meaning representation for sentences and that integrates two different kinds of updates into this representation in an interleaving manner, one kind of update for appositives and one for the at-issue part of the sentence. Section §4 takes a closer look at the behavior of clause-medial and clause-final appositive content in conversation. Section §5 extends the account and provides an analysis of appositive-content projection past sentential operators, a task that has proven difficult for previous unidimensional accounts. The basic idea is that the appositive vs. at-issue distinction should be treated as a special kind of modal subordination: the two types of content are similar to the actual vs. hypothetical possibilities involved in modal subordination discourses. Section §6 concludes.

2 Phenomena crossing the boundary

Based on data from anaphora and presupposition, [Nouwen \(2007\)](#) and [Amaral et al \(2007\)](#) have argued that appositive content cannot be wholly separate from at-issue content, as multidimensionality would allow. In this section, we show that these arguments hold for cross-sentential anaphora and presuppositions quite systematically. Furthermore, we present novel data showing a parallel pattern for NP ellipsis and VP ellipsis.²

Finally, we show that all three of these processes operate freely in *both directions*, i.e., both in the at-issue \Rightarrow appositive direction and in the reverse, appositive \Rightarrow at-issue direction. The bi-directionality of boundary-crossing phenomena serves as crucial evidence in favor of an *incremental and interleaved* account like the one in §3, in which the parts of the main and appositive clauses are interpreted left-to-right in the order in which they are uttered – as opposed to a *static/non-incremental and non-interleaved* account where the full appositive content is somehow extracted and interpreted separately from and (often) prior to the at-issue content.

2.1 Presupposition

Presupposition resolution with a variety of triggers is possible both in (a) the appositive \Rightarrow at-issue direction and (b) the at-issue \Rightarrow appositive direction. This includes both strong and weak presupposition triggers in the sense of [Abusch \(2010\)](#): *either* in (3a) and (3b) below, *too* in (4a) and (4b), the restorative reading of *again* in (5a) and (5b), its non-restorative reading in (6) and, finally, the aspectual verb *stop* in (7a) and (7b). The presupposition triggers appear in small caps and their satisfying antecedents are underlined.

- (3) a. John, who wouldn't talk to Mary, wouldn't talk to SUSAN EITHER.
b. John wouldn't talk to Mary, who wouldn't talk to HIM EITHER.
- (4) a. John, who saw Mary, saw SUSAN TOO.
b. John saw Mary, who saw HIM TOO.
- (5) a. John, who has been sick, is now HEALTHY AGAIN.
b. The window will be opened by Mary, who will then CLOSE IT AGAIN.

²While ellipsis in general crosses the boundary freely, a systematic exception to this is Sluicing, e.g., *John, who once killed a man in cold blood, wondered who *(it was)*. See [AnderBois \(2010\)](#) for discussion and an account of this exception.

- (6) Suppliers produce a garment, double the cost and sell it to a retailer, who DOUBLES THE COST AGAIN and sells it to a consumer. (COCA)
- (7) a. John, who is now building a sandcastle, will STOP soon.
b. The sandcastle was only halfway built by John, who had suddenly STOPPED.

Finally, perhaps unsurprisingly, a presupposition introduced in one appositive can be *cross-sententially* retrieved in a subsequent appositive, as shown in (8) below.

- (8) “Joe!” exclaims a young woman, who jumps in the air and throws her arms around him.
“Joe! Joe! It’s really you!” cheers a second young woman, who HUGS HIM TOO. (COCA)

2.2 Anaphora

Various types of anaphora exhibit the same kind of bi-directional boundary crossing behavior. Singular anaphora is exemplified in (9a) below, where the pronoun HER can retrieve an antecedent in the appositive, and in (9b), where the pronoun and the presupposition retrieve antecedents in the main clause. Similarly, plural anaphora to (certain) quantifiers is felicitous, both in the at-issue⇒appositive direction (10a) and vice-versa (10b). Finally, modal anaphora and subordination is exemplified in (11a) and (11b) and quantificational subordination in (12a) and (12b).

- (9) a. John, who had been kissed by Mary/a woman, kissed HER TOO.
b. John kissed Mary, who kissed HIM TOO.
- (10) a. Every speaker, all of THEM PhD students, gave a great talk.
b. Jones, who graded each student’s final paper, gave THEM detailed feedback.
- (11) a. John, who might give a presentation, WOULD use slides. Bill WOULD just use the board.
b. John might punch Jorge, who WOULD punch John back.
- (12) a. Mary, who courts a semanticist at every conference party, ALWAYS dances with HIM.
b. Mary courts a semanticist at every conference party, where she ALWAYS dances with HIM.

2.3 Ellipsis

Data from NP- and VP-Ellipsis (NPE/VPE) point in the same direction. Since NPE arguably does not require a linguistic antecedent (i.e., it is a type of deep anaphora in the sense of [Hankamer & Sag 1976](#)), we might expect the examples in (13) and (14) to be possible regardless of the status of appositive content.

- (13) a. Melinda, who won three games of tennis, lost because Betty won SIX.
b. Melinda lost three games of tennis to Betty, who lost SIX to Jane.
- (14) a. The 1980's were dominated by the Lakers, who won five championships, and by the Boston Celtics, who won THREE. (COCA)
b. "When we've got four or five guys hitting threes," said guard Pat Bradley, who made THREE, ... (COCA)

But we find examples of VPE – a type of surface anaphora, requiring a *linguistic* antecedent – in both directions, as shown below.

- (15) a. Mr. Gore at first believed the president, and even defended him to Tipper and his daughters, who DID NOT. (COCA)
b. So Lalonde, who was the one person who could deliver Trudeau, DID. (COCA)

As expected, we find the usual kind of strict/sloppy ambiguities, exemplified in (16a)-(16b) below. For example, (16a) can be interpreted as saying that Jane was told to help Mary's sister (strict reading) or Jane's (sloppy reading). This indicates that the appositive and at-issue components require access not only to each other's linguistic form, but also to the corresponding *semantic representations*.³

- (16) a. Mary, who doesn't help her sister, told Jane TO.
b. John, who helps people if they want him to, kisses them even if they DON'T.

Finally, the example in (17) below exemplifies VPE from one appositive to another across at-issue items.

³The force of this argument for ellipsis would be weakened if a purely *syntactic* theory was able to account for these ellipsis processes. Such a theory, however, strikes us as unlikely independently of appositives. More detailed discussion is beyond the scope of the present work but see, for example, [Merchant \(2001\)](#) for a semantic account of Sluicing and VPE and a discussion of the challenges faced by purely syntactic accounts.

- (17) I got a few quick words with Halle Berry, who looked amazing in Prada,
and Sigourney Weaver, who DIDN'T. (COCA)

2.4 Discussion

In sum, a wide variety of anaphora, presupposition, and ellipsis processes do not distinguish between appositive and at-issue content. These processes can operate in either direction, subject to linear order. We conclude, together with [Amaral et al \(2007\)](#) and [Schlenker \(2009a,b\)](#), that appositive and at-issue content is fundamentally *unidimensional* and *incremental*. There are not, for example, separate stores of discourse referents for different dimensions, nor separate stores of elliptical antecedents.

[Potts \(2005\)](#)'s semantics is a static one, not intended to provide an account of anaphoric processes of these sorts. But it might be possible to specify the discourse behavior of the different dimensions in ways that would account for the behavior of presuppositions, for example. In fact, [Potts \(2005\)](#)'s prose suggests that appositives are similar in some respects to independent assertions and this observation, coupled with a suitable account of presupposition resolution, might account for the facts listed above. At the same time, however, Potts also describes appositives as “deemphasized material” (p. 33) and given the role that *salience* is often assumed to play in anaphoric processes, we might reasonably expect appositives to behave differently than independent assertions. All of this is to say that there might be ways of formalizing multidimensionality that could account for certain anaphoric phenomena (most clearly presupposition), but the system in [Potts \(2005\)](#) itself does not do so.

Moreover, as we will discuss in detail in §3.1, simply specifying the discourse behavior of the two dimensions, i.e., their behavior at sentence and text level, cannot be a fully general account of these anaphoric connections. The problem arises in configurations where a given dimension both introduces an antecedent for future anaphora and includes an anaphoric element which is retrieved from outside that content – and all this happens at subsentential level. The clearest case of this would be an appositive which introduces an anaphoric antecedent and contains an element that is anaphoric to prior at-issue content. We schematize such a case in (18), where superscripts indicate the introduction of a discourse referent and subscripts the anaphoric retrieval of one.

- (18) AT-ISSUEⁱ ... APPOSITIVE_i^j ... AT-ISSUE_j

Building up separate semantic representations for the two dimensions

and specifying how each behaves will not be enough to capture this sort of interleaving. The two dimensions are constantly interacting at subsentential level, and it is hard to see how the two can be composed in entirely separate ways. Instead, in order to capture the anaphoric links in (18), each update must be interpreted incrementally from left to right.⁴ The potential for these sorts of complex interactions is exactly what we meant in the introduction when we described the systematicity of anaphoric connections as ‘underappreciated’. If we limited our attention to the anaphoric chain indicated by *j* in (18), we might well be able to, say, claim that appositives are assertions which are interpreted prior to the at-issue content. That is, simply specifying the discourse-level behavior of each separate dimension would provide a satisfactory account. These sorts of complex interleavings, however, make clear that the construction of the two contents cannot possibly be wholly separate.⁵

3 The Account

The central challenge for an analysis of appositives is how to reconcile this robust lack of anaphoric separation with the data and intuition motivating Potts (2005)’s multidimensional account. That is, how can we capture the interpretive contrast between (1) and (2), repeated in (19) and (20) below, under a single semantic representation?

(19) John, who played tennis with a woman, played golf with her too.

(20) John played tennis with a woman and played golf with her too.

Our basic idea is that appositive content and at-issue content differ principally in *how* they enter the Common Ground (CG)/Context Set (CS). At-issue assertions are *proposals* to update the CG/CS (as argued in detail by Farkas & Bruce 2010), canonically subject to acceptance or rejection by other conversational participants. Appositive content, on the other hand, is *imposed*⁶ on the common ground, with little room for negotiation (responses to appositives will be discussed in detail in §4).

⁴For simplicity’s sake, we assume that incremental update takes place in a purely left-to-right fashion, ignoring the possibility for cataphoric links across the appositive/at-issue meaning boundary.

⁵This observation holds regardless of how the relationship between the appositive anchor and the relative pronoun is established. Even if the relative pronoun and anchor are related syntactically, rather than anaphorically, examples with other anaphoric elements, like (24) below, still illustrate this point.

⁶Thanks to Floris Roelofsen for suggesting this term.

This basic distinction can be phrased in quasi-Stalnakerian terms as follows. Let us take the designated propositional variable p^{cs} to store the current CS. Then, the at-issue component puts forth a proposal, p^{issue} , to update the CS by restricting possible future contexts to those that have non-empty intersections with p^{issue} , namely $p^{cs} \cap p^{issue}$. The addressee can accept or reject this proposal. If accepted, the CS is updated by assigning a new value to the variable p^{cs} , namely the intersection of the old p^{cs} with p^{issue} , which we formalize as shown below. We use $:=$ to indicate (re)assignment of values to variables.

$$(21) \quad p^{cs} := p^{cs} \cap p^{issue}$$

In contrast, an appositive *imposes* – as opposed to *proposes* – an update on the CS with its propositional content p^{appos} . In the terms of [Farkas & Bruce \(2010\)](#), an appositive is not placed on the discourse table, i.e., the update below happens without negotiation (at least, without the regular kind of negotiation associated with at-issue content) and more or less automatically:

$$(22) \quad p^{cs} := p^{cs} \cap p^{appos}$$

Furthermore, in this simple version, appositive updates are always required to precede at-issue updates, as shown below. The semicolon ; formalizes dynamic conjunction, which requires us to update with the left conjunct first and with the second conjunct only after that.

$$(23) \quad p^{cs} := p^{cs} \cap p^{appos}; p^{cs} := p^{cs} \cap p^{issue}$$

Thus far, the approach is similar to [Murray \(2009a\)](#)/[Murray \(2009b\)](#)'s approach to evidentials in Cheyenne. In these works, an evidential directly updates the common ground prior to the at-issue proposal (whether it is an assertion or a question).

3.1 Two types of updates

While such an account in terms of *separate and complete sentence-level updates* that are sequenced in a particular way may work for evidentials, it fails to account for the phenomena discussed in section §2. Consider, for example, (24) below.

- (24) John, who nearly killed a woman with his car, visited her in the hospital.

On one hand, the content of the appositive cannot be determined independently of the at-issue component because the pronoun *his* in the appositive is anaphoric to the proper name *John* in the main clause. On

the other hand, the content of the main clause cannot be determined independently of the appositive component because the pronoun *her* in the main clause is anaphoric to the indefinite *a woman* in the appositive. Thus, we need to capture the ‘crossed’ anaphoric connections between the appositive and the main clause to properly determine their propositional contents p^{appos} and p^{issue} . That is, the appositive and at-issue updates need to be interwoven to resolve anaphora and presupposition, but we still need to distinguish the distinct nature of *at-issue proposals* and *appositive impositions*.

As a first attempt, suppose we follow Heim (1982) and represent the CS by means of a designated world variable w^{cs} . At any point in discourse, the information state at that point consists of all the variable assignments that are still live options. The CS consists of the worlds assigned to the variable w^{cs} by all these assignments and is encoded implicitly by the rows in (25):

$$(25) \begin{array}{l} w^{cs} \\ \boxed{\mathbf{w}_1} \\ \\ w^{cs} \\ \boxed{\mathbf{w}_2} \\ \\ w^{cs} \\ \boxed{\mathbf{w}_3} \end{array}$$

A sequence of (at-issue) updates is easy to capture – every update eliminates more and more assignments and, therefore, worlds associated with the variable w^{cs} .

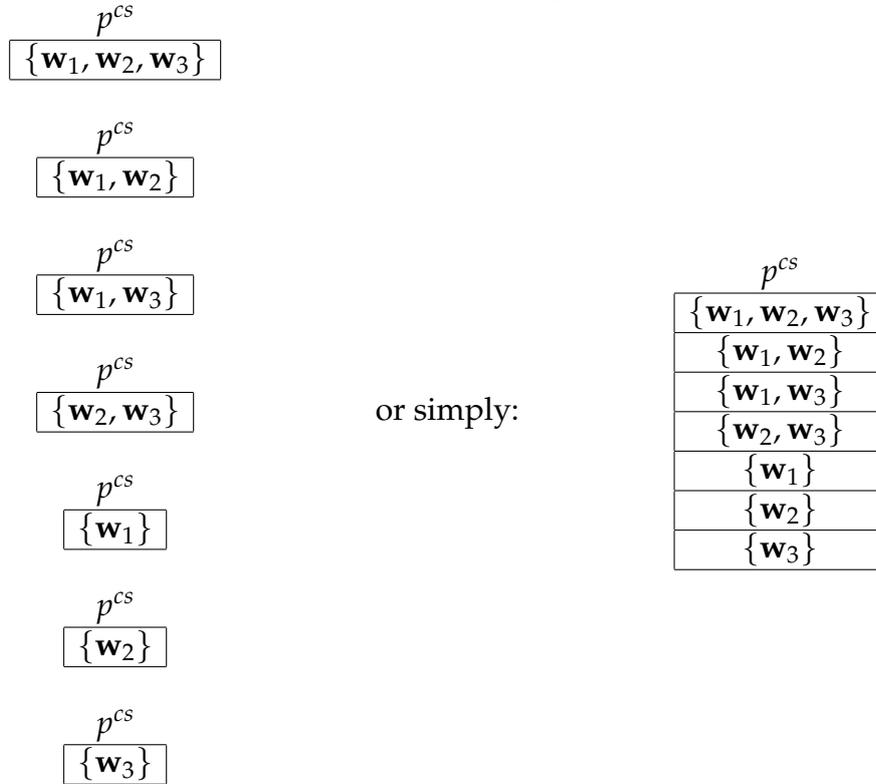
(26) John^x nearly killed a^y woman with his_x car. He_x visited her_y in the hospital.

$$(27) \begin{array}{l} w^{cs} \\ \boxed{\mathbf{w}_1} \\ \\ w^{cs} \\ \boxed{\mathbf{w}_2} \\ \\ w^{cs} \\ \boxed{\mathbf{w}_3} \end{array} \xrightarrow{\text{John}^x \text{ nearly killed a}^y \text{ woman in } w^{cs}} \begin{array}{l} w^{cs} \quad x \quad y \\ \boxed{\mathbf{w}_1 \mid \text{john} \mid \text{woman}_1} \\ \\ w^{cs} \quad x \quad y \\ \boxed{\mathbf{w}_2 \mid \text{john} \mid \text{woman}_2} \end{array}$$

$$\xrightarrow{\text{He}_x \text{ visited her}_y \text{ in } w^{cs}} \begin{array}{l} w^{cs} \quad x \quad y \\ \boxed{\mathbf{w}_1 \mid \text{john} \mid \text{woman}_1} \end{array}$$

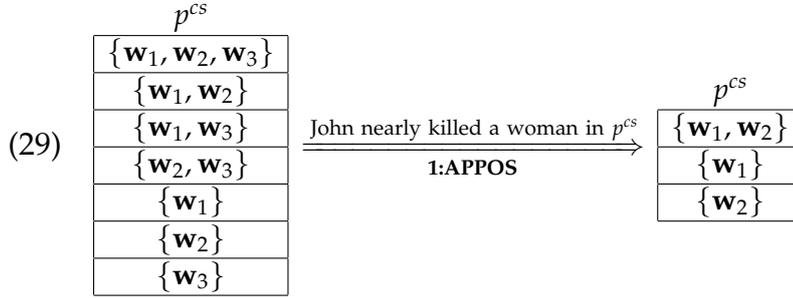
But if we model the CS by means of a world variable w^{cs} , there is only one way to update the CS: we eliminate worlds by eliminating assignments, incrementally restricting the CS. While both at-issue and appositive updates restrict the CS to one of its subsets, they do so in different ways, so we need *two different ways* to select subsets of the CS. We will accomplish this in three steps.

The first step is to model the CS by means of a propositional variable p^{cs} that stores the current CS and all its non-empty subsets (Gunlogson 2001 makes a similar proposal). For example, if the current CS is the set of worlds $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$, the current information state, i.e., the set of assignments that are still live options in discourse, is as represented on the left-hand side below. For readability, we will graphically depict such information states as shown on the right-hand side.



The second step is taking appositives to contribute eliminative, Heim-style updates. For example, suppose that John nearly killed a woman with his car only in worlds \mathbf{w}_1 and \mathbf{w}_2 . We will then eliminate all the assignments that assign to p^{cs} at least one world in which this is not true. That is, we will eliminate all the assignments that assign to p^{cs} a set that includes world \mathbf{w}_3 . For ease of reference, we will generally number the updates and indicate whether they are appositive or at-issue, e.g., the update below is identified as **1:APPOS**, i.e., update number 1, contributed by the appositive part of the sentence/discourse.

- (28) John, who nearly killed a woman with his car, visited her in the hospital.



The appositive update is a *test* on the variable p^{cs} , just as *woman*(y) is a test on the variable y , and contributes new information about the values of that variable.

The third and final step is to have at-issue updates put forth a proposal p (or p^{issue} , but we generally omit the superscript for readability) to update the CS by restricting the sets of worlds p^{cs} to a subset $p \subseteq p^{cs}$. If the proposal is accepted, the new context set becomes p , provided that p is non-empty.

For example, suppose that John visited a woman in the hospital only in worlds \mathbf{w}_1 and \mathbf{w}_3 . Then, as shown in (31) below, the proposal p will store both these worlds or only one of them after the update **1:AT-ISSUE**. In more detail, consider the case in which p^{cs} is assigned the set of worlds $\{\mathbf{w}_1, \mathbf{w}_2\}$. Since the proposal p has to be a subset of the Context Set p^{cs} , the only possible assignment to p is the singleton set $\{\mathbf{w}_1\}$. If on the other hand p^{cs} is assigned the set of worlds $\{\mathbf{w}_1, \mathbf{w}_3\}$, p can be assigned $\{\mathbf{w}_1, \mathbf{w}_3\}$, or $\{\mathbf{w}_1\}$, or $\{\mathbf{w}_3\}$.

Finally, if the proposal p is accepted, the variable p^{cs} is assigned the same values as p , and the power set previously stored in p^{cs} is shrunk to a (much) smaller power set, as shown after the update **2:AT-ISSUE** in (31) below.⁷

- (30) John visited a woman in the hospital.

⁷Technically, the output of the **1:AT-ISSUE** update in (31) should also have assignments that assign the empty set of worlds \emptyset to p – one such assignment for each set of worlds assigned to p^{cs} in the initial input information state. The ‘provided that’ clause in the above description of the third step (‘the new context set becomes p , provided that p is non-empty’) ensures that such rows will be eliminated if p is accepted, ruling out absurd updates, so we omit them to begin with for readability.

$$(31) \quad \begin{array}{c} p^{cs} \\ \hline \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\} \\ \hline \{\mathbf{w}_1, \mathbf{w}_2\} \\ \hline \{\mathbf{w}_1, \mathbf{w}_3\} \\ \hline \{\mathbf{w}_2, \mathbf{w}_3\} \\ \hline \{\mathbf{w}_1\} \\ \hline \{\mathbf{w}_2\} \\ \hline \{\mathbf{w}_3\} \end{array} \xrightarrow[\text{1:AT-ISSUE}]{p \subseteq p^{cs} \wedge \text{John visited a woman in } p} \begin{array}{cc} p^{cs} & p \\ \hline \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\} & \{\mathbf{w}_1, \mathbf{w}_3\} \\ \hline \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\} & \{\mathbf{w}_1\} \\ \hline \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\} & \{\mathbf{w}_3\} \\ \hline \{\mathbf{w}_1, \mathbf{w}_2\} & \{\mathbf{w}_1\} \\ \hline \{\mathbf{w}_1, \mathbf{w}_3\} & \{\mathbf{w}_1, \mathbf{w}_3\} \\ \hline \{\mathbf{w}_1, \mathbf{w}_3\} & \{\mathbf{w}_1\} \\ \hline \{\mathbf{w}_1, \mathbf{w}_3\} & \{\mathbf{w}_3\} \\ \hline \{\mathbf{w}_2, \mathbf{w}_3\} & \{\mathbf{w}_3\} \\ \hline \{\mathbf{w}_1\} & \{\mathbf{w}_1\} \\ \hline \{\mathbf{w}_3\} & \{\mathbf{w}_3\} \end{array}$$

$$\xrightarrow[\text{2:AT-ISSUE}]{p^{cs}=p} \begin{array}{cc} p^{cs} & p \\ \hline \{\mathbf{w}_1, \mathbf{w}_3\} & \{\mathbf{w}_1, \mathbf{w}_3\} \\ \hline \{\mathbf{w}_1\} & \{\mathbf{w}_1\} \\ \hline \{\mathbf{w}_3\} & \{\mathbf{w}_3\} \end{array}$$

Thus, both at-issue and appositive updates contribute new information, i.e., they both restrict the values assigned to the variable p^{cs} . The appositive update is eliminative and targets the variable p^{cs} directly. In contrast, the at-issue update contributes a proposal p that is a subset of the CS variable p^{cs} . If the proposal is accepted, the set of worlds assigned to p is also assigned to p^{cs} .

Importantly, we can freely interleave these two ways of restricting the CS. Basically, we make use of the fact that dynamic semantics keeps track of two kinds of interwoven information that can be updated simultaneously:

- i. the *factual, propositional* information stored by the sets of worlds that are assigned as values to propositional variables p, p', \dots – roughly speaking, this is used to formalize the new information contributed by the at-issue update;
- ii. the *anaphoric* information stored by variable assignments and incrementally constrained in discourse – roughly speaking, this is used to formalize the new information the appositive update imposes.

Together, the two updates can be represented in the linear order in which they occur, as shown below. Assume that John nearly killed $woman_1$ and $woman_2$ in world \mathbf{w}_1 and, also, in world \mathbf{w}_2 . Further assume that there is no near-killing in world \mathbf{w}_3 . Finally, assume that John only visited $woman_1$ in world \mathbf{w}_1 .

- (32) John^x, who_x nearly killed a^y woman with his_x car, visited her_y in the hospital.

(33)

p^{cs}
$\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$
$\{\mathbf{w}_1, \mathbf{w}_2\}$
$\{\mathbf{w}_1, \mathbf{w}_3\}$
$\{\mathbf{w}_2, \mathbf{w}_3\}$
$\{\mathbf{w}_1\}$
$\{\mathbf{w}_2\}$
$\{\mathbf{w}_3\}$

 $\xrightarrow[1:AT-ISSUE]{p \subseteq p^{cs} \wedge x = \text{JOHN}}$

p^{cs}	p	x
$\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$	$\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$	john
$\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$	$\{\mathbf{w}_1, \mathbf{w}_2\}$	john
$\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$	$\{\mathbf{w}_1, \mathbf{w}_3\}$	john
	...	
$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_1, \mathbf{w}_2\}$	john
$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_1\}$	john
$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_2\}$	john
$\{\mathbf{w}_1, \mathbf{w}_3\}$	$\{\mathbf{w}_1, \mathbf{w}_3\}$	john
$\{\mathbf{w}_1, \mathbf{w}_3\}$	$\{\mathbf{w}_1\}$	john
$\{\mathbf{w}_1, \mathbf{w}_3\}$	$\{\mathbf{w}_3\}$	john
$\{\mathbf{w}_2, \mathbf{w}_3\}$	$\{\mathbf{w}_2, \mathbf{w}_3\}$	john
$\{\mathbf{w}_2, \mathbf{w}_3\}$	$\{\mathbf{w}_2\}$	john
$\{\mathbf{w}_2, \mathbf{w}_3\}$	$\{\mathbf{w}_3\}$	john
$\{\mathbf{w}_1\}$	$\{\mathbf{w}_1\}$	john
$\{\mathbf{w}_2\}$	$\{\mathbf{w}_2\}$	john
$\{\mathbf{w}_3\}$	$\{\mathbf{w}_3\}$	john

 $\xrightarrow[2:APPOS]{\text{who}_x \text{ nearly killed a}^y \text{ woman in } p^{cs}}$

p^{cs}	p	x	y
$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_1, \mathbf{w}_2\}$	john	woman ₁
$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_1, \mathbf{w}_2\}$	john	woman ₂
$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_1\}$	john	woman ₁
$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_1\}$	john	woman ₂
$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_2\}$	john	woman ₁
$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_2\}$	john	woman ₂
$\{\mathbf{w}_1\}$	$\{\mathbf{w}_1\}$	john	woman ₁
$\{\mathbf{w}_1\}$	$\{\mathbf{w}_1\}$	john	woman ₂
$\{\mathbf{w}_2\}$	$\{\mathbf{w}_2\}$	john	woman ₁
$\{\mathbf{w}_2\}$	$\{\mathbf{w}_2\}$	john	woman ₂

 $\xrightarrow[3:AT-ISSUE]{x \text{ visited } y \text{ in } p}$

p^{cs}	p	x	y
$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_1\}$	john	woman ₁
$\{\mathbf{w}_1\}$	$\{\mathbf{w}_1\}$	john	woman ₁

 $\xrightarrow[4:AT-ISSUE]{p^{cs} = p}$

p^{cs}	p	x	y
$\{\mathbf{w}_1\}$	$\{\mathbf{w}_1\}$	john	woman ₁

3.2 Appositives vs. presuppositions

Like appositives, a presupposition targets the input CS through the dref p^{cs} directly. Neither presuppositions nor appositives are part of the at-issue proposal p to update the CS. However, there is a fundamental difference between them. Aside from their non-negotiability, appositives are ordinary *updates* of the current information state (i.e., the current CS in Stalnakerian terms). A felicitous use of an appositive, then, is truth-conditionally *informative*.

In contrast, presuppositions are *constraints* or *preconditions* on the current information state/CS. They are required to be satisfied *throughout the entire input information state/CS*. That is, any assignment g in the input information state has to satisfy the presupposition.⁸ Presupposed meaning is taken for granted and anaphorically links the non-presupposed (at-issue and/or appositive) meaning with its context of interpretation.

Presuppositions which are not met throughout the input CS can, of course, be *accommodated*, but they can also cause infelicity, i.e., presupposition failure. There is no analogous phenomenon of “apposition failure” since appositive content is not presumed to be known.⁹ Accommodation, however, is an exceptional means of repairing the input information state/CS, rather than being the general case. Thus, we take anaphoricity to be an important part of what it means to presuppose something, following [van der Sandt \(1992\)](#) and others (e.g., see [Kamp 2001](#) for a more recent discussion).

Given the characterization of presuppositions as preconditions on the CS – as opposed to appositives, which are tests – we correctly expect that appositives themselves may contribute presuppositions. For example, the appositive update of (34) presupposes that John has a Ferrari and that he washed it prior to today.

- (34) John, who washed his Ferrari again today, is completely obsessive.

Finally, note that presuppositions in an appositive can be satisfied by at-issue content, as in (9b) above (*John kissed Mary, who kissed HIM TOO*).

⁸We can formalize this along the lines of [van der Sandt \(1992\)](#); see also the discussion in [Krahmer \(1998\)](#), Ch. 6.

⁹The only way apposition can fail is if the appositive content is incompatible with the CS. In this case, no proposal, not even a tautology can be accepted without ending up in the absurd state. Given that rational, truth-seeking speakers should avoid the empty CS at all costs, using an appositive that is incompatible with the CS should put the conversation in crisis. Note that the kind of accommodation used to resolve presupposition failure is not available here because the issue is not whether the CS can be shrunk so that some φ is satisfied throughout, but that φ is not satisfiable at any world in the CS.

This occurs with presuppositions that are hard to accommodate (*too*), so we predict that the appositive forces the acceptance of the at-issue proposal prior to the appositive update. Thus, proposals to update the common ground do not come only in sentence/clause-sized chunks. This is very much in line with [Clark & Schaefer \(1989\)](#) and others, who observe that discourse negotiation mostly takes place at sub-clausal level; see also [Koev \(2012\)](#) for a similar proposal couched in a closely related dynamic system.

3.3 Dynamic proposals

We formalize these two types of updates in an extension of Dynamic Predicate Logic ([Groenendijk & Stokhof 1991](#)). Our models have the same structure as the ones for classical, static first-order modal logic, i.e., they consist of the disjoint domains of individuals \mathfrak{D} and possible worlds \mathfrak{W} , and the basic interpretation function \mathcal{I} that assigns a subset of \mathfrak{D}^n to any n -ary relation R relative to any world \mathbf{w} , i.e., $\mathcal{I}_{\mathbf{w}}(R) \subseteq \mathfrak{D}^n$ (see the appendix for the complete formal system).

We have variables over individuals (x, y, \dots) , worlds (w, w', \dots) and propositions/sets of worlds (p, p', p^{cs}, \dots) , and the usual inventory of non-logical constants: individual constants (JOHN, \dots), properties (WOMAN, \dots), binary relations (VISIT, \dots) etc. Formulas are interpreted relative to a pair of assignments $\langle g, h \rangle$, i.e., they denote binary relations between an input assignment g and an output assignment h . In particular, dynamic conjunction – which we choose to symbolize as ‘ \wedge ’ rather than ‘ $;$ ’ – is interpreted as relation composition:

$$(35) \quad \llbracket \varphi \wedge \psi \rrbracket^{\langle g, h \rangle} = \mathbb{T} \text{ iff} \\ \text{there exists a } k \text{ such that } \llbracket \varphi \rrbracket^{\langle g, k \rangle} = \mathbb{T} \text{ and } \llbracket \psi \rrbracket^{\langle k, h \rangle} = \mathbb{T}$$

New variables are introduced by means of random assignment formulas $[x]$, $[p]$, etc.

$$(36) \quad \llbracket [v] \rrbracket^{\langle g, h \rangle} = \mathbb{T} \text{ (for any variable } v) \text{ iff} \\ g \text{ differs from } h \text{ at most with respect to the value assigned to } v, \\ \text{i.e., for any variable } v' \text{ s.t. } v' \neq v, \text{ we have that } g(v') = h(v')$$

Lexical relations relativized to propositional variables p, p', p^{cs}, \dots are distributively interpreted relative to these propositional variables. For example:

$$(37) \quad \begin{array}{l} \text{a. } \llbracket \text{WOMAN}_p(x) \rrbracket^{\langle g, h \rangle} = \mathbb{T} \text{ iff} \\ \quad g = h \text{ and for all worlds } \mathbf{w} \in h(p), h(x) \in \mathcal{I}_{\mathbf{w}}(\text{WOMAN}) \\ \text{b. } \llbracket \text{VISIT}_p(x, y) \rrbracket^{\langle g, h \rangle} = \mathbb{T} \text{ iff} \\ \quad g = h \text{ and for all worlds } \mathbf{w} \in h(p), \langle h(x), h(y) \rangle \in \mathcal{I}_{\mathbf{w}}(\text{VISIT}) \end{array}$$

3.4 Formalizing discourse reference across the appositive/at-issue boundary

Sentence (38) (repeated from above) is represented as in (39) below.

- (38) John^x, who nearly killed a^y woman with his_x car, visited her_y in the hospital.
- (39) a. **New proposal:** $[p] \wedge p \subseteq p^{cs} \wedge$
 b. **Issue:** $[x] \wedge x = \text{JOHN} \wedge$
 c. **Appositive:** $[y] \wedge \text{WOMAN}_{p^{cs}}(y) \wedge \text{NEARLY-KILL}_{p^{cs}}(x, y) \wedge$
 d. **Issue:** $\text{VISIT}_p(x, y) \wedge$
 e. **Proposal accepted:** $[p^{cs}] \wedge p^{cs} = p$

The formula in (39a) introduces the proposal to update the CS: we introduce a new variable $p \subseteq p^{cs}$ containing worlds satisfying the subsequent at-issue update. The formulas in (39b) and (39d) are the two at-issue updates and the formula in (39c) is the appositive update. They instruct us to introduce a new variable x whose value is John and comment that x nearly killed a woman y and x visited y . The appositive nature of the update in (39c) is captured by the fact that the appositive content is interpreted relative to p^{cs} rather than relative to the new proposal p . The following update in (39d), i.e., $\text{VISIT}_p(x, y)$, is part of the at-issue proposal, so it is interpreted relative to p .

In terms of composition, the most straightforward way to implement this is by using left and right comma operators following [Nouwen \(2007\)](#). While for Nouwen these operators toggle back and forth between two Potts-style dimensions, for us they toggle back and forth between p^{cs} and p . That is, a left comma operator indicates that the immediately following content should update p^{cs} , while a right comma operator toggles back to updating p itself. Since the left comma operator picks out p^{cs} directly, this accounts for the fact that appositives, even when inside the scope of attitude reports, are (generally) speaker-oriented; we return to this and related issues in section §5 below.¹⁰

We do not explicitly represent the comma operator in (39) above, and throughout this paper, to keep the exposition and formulas simpler and more readable. Instead, we will assume that various parts of the sentence/discourse are already correctly indexed with the relevant propositional discourse referents (drefs). From this perspective,

¹⁰Following arguments in [Harris & Potts \(2009\)](#), we take cases of non-speaker orientation to be the result of a pragmatically-driven perspective shift, separate from the semantics of appositives. We can accommodate such perspective shifts in our system as a special kind of modal anaphora – see for example [Brasoveanu \(2010\)](#) and references therein for a dynamic account of related cases of modal anaphora in a dynamic system similar to the present one.

Nouwen’s comma operator is just the formalization of a particular constraint on how appositives vs. main clauses are indexed by/relativized to propositional drefs that are available in their discourse context.

One consequence of this worth pointing out is that we predict that an appositive inside of another appositive, like the one underlined in (40) below,¹¹ will exhibit the same behavior as any other since both will be indexed by p^{cs} . To our knowledge, this is a welcome prediction since we are unaware of any special semantic/discourse properties of such appositives.¹²

- (40) I will hire John, who my boss, who just called me, praises rather warmly.

Finally, (39e) contributes the proposal to update the CS variable p^{cs} by resetting it to p . Despite the possible non-maximality of the set of worlds p , the Stalnakerian CS (which is the maximal set of worlds compatible with both the previous CS and the at-issue proposal) will always be recoverable: after the update in (39d), there will be an output assignment h such that $h(p)$ contains the *maximal* set of worlds in the current CS that satisfy the at-issue relation $VISIT_p(x, y)$. Therefore, the output *information state*, i.e., the whole set of output variable assignments that are still live candidates in discourse, will necessarily contain an assignment storing this maximal set of worlds, which is the new CS in Stalnaker’s sense.¹³

Since the account is formally *unidimensional*, ellipsis processes like VPE can be accounted for straightforwardly. One way to accomplish this is to extend the dynamic framework with discourse referents/variables for properties along the lines of Hardt (1999) and Stone & Hardt (1999). Just as the indefinite *a woman* in the appositive in (38) introduces a discourse referent, the antecedent VP does too. Retrieving the missing VP at the ellipsis site is similarly parallel to the way in which the pronoun *her* in the main clause anaphorically retrieves the discourse referent introduced by the indefinite.

¹¹Thanks to Philippe Schlenker for bringing such cases to our attention and for the example in (40).

¹²While we believe that Potts (2005) makes essentially the same prediction in this regard, it would be easy to define a version of his COMMA operator that would assign such appositives to a third dimension, call it CI_2 . The present framework does not seem to allow for such a possibility.

¹³In other words: let H be the set of output variable assignments obtained after the sequence of updates in (39) above. The new CS in Stalnaker’s sense is the maximal set of worlds in the set $\{h(p^{cs}) : h \in H\}$ or, alternatively, the set of worlds $\bigcup_{h \in H} h(p^{cs})$. Given the way we defined the CS, which is closed under subsets and unions, and the fact that we will deal with formally ‘well-behaved’ updates in this paper, there will usually be a particular individual assignment $h^* \in H$ that will actually store this maximal CS set, i.e., $h^*(p^{cs}) = \bigcup_{h \in H} h(p^{cs})$.

Before proceeding, it is worth briefly revisiting the question of ‘multidimensionality’ and contrasting the formal system here with Potts (2005) in this respect. As mentioned in the introduction, multidimensionality has usually been discussed with respect to classical static semantics, and each dimension in a multidimensional static system is thought of in terms of truth conditions/truth values. Moving to a semantics where the basic notion of meaning is information update/context change potential instead, it is not immediately clear whether the present account should be classified as multidimensional or not. On one hand, at-issue information and appositive information update different things – p and p^{cs} , respectively – just as for Potts (2005), they update different members of a tuple.

On the other hand, our semantics produces a single sequence of incremental updates rather than separately building an at-issue update and an appositive update. In this sense, the current account is plainly unidimensional: there is only one semantic representation that is incrementally built and a single information state that is incrementally updated. It happens to be the case that the semantic representation updates and tests multiple propositional discourse referents, i.e., it targets different aspects/components of the information state. But this property is not at all exceptional. In fact, it’s the rule: a simple sentence like *A lady ordered a martini*, with an indefinite in subject position and another in object position, updates and tests multiple distinct individual discourse referents.

Our simultaneous, interleaved updates of propositional discourse referents are admittedly more complex than the updates contributed by such simple sentences, but they do not fundamentally enhance the underlying semantic framework. Ultimately, whether our account is taken to be ‘multidimensional’ or not is a matter of terminology. The important point to remember is that the ability of our account to both interleave and keep distinct at-issue and appositive updates is the result of basic architectural properties of even the simplest dynamic semantic frameworks.

4 Appositives in discourse

The account proposed in §3 treats appositives as updates which are *imposed* on the common ground. In this section, we explore the ways in which appositive content behaves in discourse, showing how they follow from this characterization.

4.1 Proposals and Questions Under Discussion

One central difference between at-issue and appositive content is the relative inability of the latter to interact with the question under discussion (QUD).¹⁴ While apparent examples of appositive questions occur in *written* English, as in (41-42), they are quite infelicitous in *spoken* English. That is, appositive content cannot explicitly introduce a new QUD the way that at-issue content can.¹⁵

- (41) This unknown person fell in love with Carlos, and, in a moment of rage and jealousy (who knew what Carlos felt?) beat Warren to death. (COCA)
- (42) The producers are on vacation in Hawaii. Larry (who's Larry?) is on the golf course and can't be reached. (COCA)

Parallel to this, appositive content cannot readily resolve an existing QUD. We see this plainly in the contrast between (43) and (44) below: while the appositive content in (43) clearly resolves the question, it's appositive nature makes it an infelicitous answer.¹⁶

- (43) a. Who had prostate cancer?
b. ??Tammy's husband, who had prostate cancer, was being treated at the Dominican Hospital.
- (44) a. Who was being treated at the Dominican Hospital?

¹⁴For the notion of *question under discussion*, see Roberts (1996), Ginzburg (1996), Büring (2003) and Farkas & Bruce (2010) among others.

¹⁵Chris Potts (p.c.) mentions naturally occurring examples like (i-ii) as apparent counterexamples. These examples are very interesting, but we believe that they do not establish a new QUD parallel to main clause questions, but rather serve other discourse purposes. Many of the examples one can find seem to be metalinguistic in nature (e.g., (ii)), others seem to be rhetorical questions or other cases of *mention* rather than *use*.

- (i) In many States, regulation of this product has fallen between the cracks of different regulatory agencies – is it insurance or managed care? – leaving consumers without the protections they need.
- (ii) Jack, the middle class in this country will lament the exorbitant (or is that extorted?) profits by ExxonMobil and other big oil, drug and insurance companies [the CNN interview show *The Situation Room*]

Example (i) could also be characterized as metalinguistic since the interpretation of the parenthetical remark seems to be: what exactly do I mean by “fallen between the cracks ...”? I mean that we cannot really answer the question “is it insurance or managed care?”.

¹⁶The picture is a bit less clear for clause/sentence-final appositives as noted by several reviewers. See §6 for discussion of this and other ways in which final appositives sometimes diverge from medial ones.

- b. Tammy’s husband, who had prostate cancer, was being treated at the Dominican Hospital.

These facts follow directly from our characterization of appositive content as not being placed on the table, in the terms of [Farkas & Bruce \(2010\)](#). The table is not only where the acceptance/rejection of assertions take place, it is also where the QUD stack is managed. One central claim in [Farkas & Bruce \(2010\)](#) – supported by the parallels between responses to at-issue assertions and polar questions – is that serving as a *proposal* to update the CS intrinsically involves the same discourse resources as managing the QUD. Since the structure of the table is what relates at-issue content to the QUD, it follows that content which imposes an update on the CS itself (as we claim for appositives) cannot interact with the QUD.

A bit more concretely, we can state the QUD generalization as in (45):

- (45) **QUD Generalization:** Only semantically at-issue content can readily address the QUD.

This generalization is closely related to [Simons et al. \(2010\)](#)’s generalization about projection and the QUD (all and only content which does not address the QUD projects). But we believe our formulation and our overall account has one crucial advantage for present purposes: a clearly defined role for the compositional semantics of at-issue and appositive content. [Simons et al. \(2010\)](#)’s generalization is a purely pragmatic one and therefore more needs to be said about why asymmetries like those in (43-44) exist. At the same time, we do not address the quite intricate presupposition data that is their empirical focus, as this would take us to far afield here. The relationships between projection, QUDs, and compositional semantics is a vast topic and we leave a more comprehensive investigation to future work.

4.2 Responding to appositive content

Another contrast between appositives and at-issue assertions is the range of possible responses an addressee can give to them. [Farkas & Bruce \(2010\)](#) show that at-issue assertions *allow* for roughly the same range of responses that polar questions *expect*. In particular, assertions readily allow for bare particle responses like *yes*, *no*, and *maybe* as in (46) below.

- (46) a. A: Sonia is coming to the party.
- b. B: Yes // No // Maybe // Perhaps.

In contrast, bare particle responses are not readily interpreted as ratifying appositive content, as the contrast in continuations between (47) and (48) below show.

- (47) a. A: Sonia, who is a terrible housemate, left the door unlocked last night.
b. B: Yeah, but she is still a good housemate.
c. B: No, but she *is* a terrible housemate.
- (48) a. A: Sonia is a terrible housemate and she left the door unlocked last night.
b. B: #Yeah, but she is still a good housemate.
c. B: #No, but she *is* a terrible housemate.

While bare particle responses do not readily target appositive content, responses that echo or expand on the appositive content are possible:

- (49) COCA, *60 Minutes*, CBS Sixty.
a. [Mr. DON FUQUA] He told me about Noah, his first-born, and how he shared his son's love of rockets. He told me about how thankful he was to have Mary, his only girl, and Luke, **who loved to have his picture taken**.
b. [SPOKESMAN] **Yeah, he always liked the camera**. He'd always smile, but he always squint his eyes and . . .
- (50) COCA, *Lisa Ling goes inside one of the world's most dangerous gangs; journalists Lisa Ling, Anderson Cooper and Brian Ross discuss some of their most memorable stories*, Ind Oprah.
a. [Mr. ANDERSEN] And there was some sense of justice, I think, for these children for me to track down this foster mother, **who really got away with outrageous behavior**.
b. [Ms. SALTZMAN] **Yeah. She got away with it**.

Denying appositives is less frequent, but also possible. A COCA search for “, who” followed by “no” in a 9-word window, revealed no clear example of appositive denial, but discourses like the following seem to be natural nonetheless:

- (51) a. He took care of his husband, who had prostate cancer.
b. No, he had lung cancer.
c. No, he took care of his brother.
- (52) a. He told her about Luke, who loved to have his picture taken.
b. No, he didn't like that at all.
c. No, he told her about Noah.

However, both of these sorts of responses rely crucially on the utterance-final nature of the appositives in question. The examples in (53-54) below, which are parallel to (51-52) above, sound quite degraded.

- (53) a. His husband, who had prostate cancer, was being treated at the Dominican Hospital.
b. ??No, he had lung cancer.
c. No, he was being treated at the Stanford Hospital.
- (54) a. Luke, who loved to have his picture taken, was his son.
b. ??No, he didn't like that at all.
c. No, Luke was his nephew.

Finally, in cases where the appositive is utterance-final, a speaker can require explicit confirmation through the use of tags such as *right*:

- (55) COCA, *Tammy Faye Messner discusses the rise and fall of the PTL*, CNN King.
a. [Mr. HAHN] And take care of your husband, who has prostate cancer, **right**?
b. [WILLIAM DALEY] **Yes**.
c. [Mr. HAHN] How's he doing?
- (56) COCA, *Charles Bryan earns right to play with Will Shortz*, NPR Weekend.
a. [ANNOUNCER] Happy Mother's Day to your mother today, who I guess is in town with you, **right**?
b. [ANNOUNCER] **That's right**, I flew her to New York as a Mother's Day gift.

Note also that in both of these cases, *right* can be felicitously replaced with an opposite polarity tag question – *doesn't he?* and *isn't she?*, respectively. Though we save developing an account of the medial vs. final contrast for future work, section §6 presents more empirical contrasts and a path towards their explanation.

5 Appositive updates and the projection problem

The previous sections presented a unidimensional, incremental semantics that distinguished two types of updates: one for appositive impositions and one for at-issue proposals. In addition to providing an account of the discourse status of both, the account allowed us to capture the fact that anaphoric processes can generally cross freely between at-issue and appositive content.

Thus far, however, we have not addressed the second sort of separation between appositive and at-issue content: the projection of the former past operators in the latter. That is, it is generally recognized that relative appositives are scopally inert. They must take wide scope relative to negation (57), modals (58), conditional antecedents (59), and attitude reports (60).

- (57) John didn't see Bill, who was hiding.
- (58) John might fight Bill, who is a professional boxer.
- (59) If John fights Bill, who is a professional boxer, he will surely lose.
- (60) John believes that Bill, who is a professional boxer, will win the fight.

The previous literature has taken one of two basic approaches to account for this apparent scopelessness. The majority approach has been to claim that appositives have special syntactic properties that explain their scopelessness (e.g., [del Gobbo 2003](#), [Demirdache 1991](#), [Nouwen 2010](#), [Schlenker 2009a,b](#)). Appositives might preferentially attach to a high adjunction site, for instance, preventing them from ever scoping under operators like negation. Combining our account with a suitable, independently motivated syntactic scoping account would capture what we take to be the core properties of appositives: their discourse behavior, anaphoric integration, and projection.

While we cannot completely discount this possibility, we are skeptical of the viability of such an approach (see also [Potts 2005](#) for more detailed arguments). In particular, we find a syntactic account of projection to be unsatisfying since it does not directly relate projection to the special behavior of appositives in discourse. In contrast, a semantic approach to projection/scopelessness has the potential to provide a unified explanation of the two phenomena. For [Potts \(2005\)](#), for example, multidimensionality is the source of both the difference in discourse status and projection.

Providing a similarly unified account within a unidimensional semantics, however, has proven problematic. The goal of this section is precisely to provide such a unified semantic account of projection within a unidimensional semantics. To accomplish this, we propose that scope-taking propositional operators such as negation, modals, conditional antecedents, and attitude predicates, introduce a new propositional discourse referent (dref) p' – a standard assumption, albeit formalized somewhat differently in other frameworks – and that the content in their scope is relativized to this new dref. This new dref's content is incorporated into the proposal p in whatever way is appropriate for that propositional operator – again, a standard assumption.

This proposal extends our account of appositives to propositional operators in a non-trivial way: both appositives and propositional operators are ultimately taken to *toggle* between the main proposal and some other propositional dref. Where appositives are special is that they toggle to the dref for the Context Set itself (i.e., p^{cs}) rather than introducing a new dref p' .

Thus, the main idea of our account is to treat the appositive vs. at-issue distinction as a special kind of modal subordination: the two types of content are similar to the actual vs. hypothetical possibilities/propositions entertained in modal subordination discourses.

The projection of appositive content, then, arises because the Nouwen-style comma operator we have implicitly assumed before toggles to p^{cs} regardless of which propositional dref is currently tested/updated. That is, the same mechanism that is responsible for the special discourse behavior of appositives – the way they update the CS – will also be responsible for projection.¹⁷

Before introducing our own account of projection past at-issue propositional operators, it is instructive to see why extending the account in §3 above with the standard dynamic semantics denotations for these operators fails to capture the correct projection behavior for appositives.

The easiest way to see the problem is to consider the case of an appositive within the scope of negation.¹⁸ Negation in dynamic frameworks is often treated as a test (it is ‘externally static’), and the test contains a universal quantifier over output variable assignments, as shown in (61) below.

$$(61) \quad \llbracket \neg\varphi \rrbracket^{g,h} = \mathbb{T} \text{ iff} \\ g = h \text{ and there is no } k \text{ s.t. } \llbracket \varphi \rrbracket^{g,k} = \mathbb{T}, \text{ i.e., equivalently, for} \\ \text{any assignment } k, \llbracket \varphi \rrbracket^{g,k} = \mathbb{F}$$

Assuming (61), whenever negation takes scope over an appositive, i.e., whenever we have updates of the form

$$\neg(\text{AT-ISSUE-CONTENT}_p(x, y, \dots) \wedge \text{APPOS-CONTENT}_{p^{cs}}(x, y, \dots)),$$

we incorrectly predict that the falsity of the appositive relative to the Context Set verifies the negated expression. That is, if no worlds in p^{cs} satisfy the appositive content, the entire negative update above will be

¹⁷Taking this idea seriously, we expect to find a variety of appositives with slightly different toggling possibilities, which is what we found when we considered final appositives in §4. The range of semantically-distinct appositive constructions is even wider, e.g., nominal appositives seem to behave differently than either medial or final appositive relative clauses, as we will discuss in §5.2.

¹⁸We would like to thank Ben George, Todor Koev and Rick Nouwen for extensive discussion of these problems.

true according to the semantic clause in (61): the conjunction within the scope of negation will be false for any possible output assignment k just because $\text{APPOSITIVE-CONTENT}_{p^{cs}}(x, y, \dots)$ is false, so the entire negative formula will be true.

More concretely, an example like (62) below is predicted to simply be true since there will be no assignment k where the formula is true (assuming that the Context Set only contains worlds where the moon is made of rock).

- (62) John hasn't visited the moon, which is made of cheese, in a space shuttle.

A syntactic solution to the problem is readily available: we can appeal to covert syntactic movement to ensure that the appositive material will not be within the semantic scope of negation. While this type of account is definitely feasible and has been pursued in the previous literature (as noted above), we will pursue here a semantic account that more directly captures the intuition that any such covert movement is ultimately driven by interpretive needs.

In particular, to account for the interaction of negation and appositives in our unidimensional dynamic framework, we propose that negation introduces a new propositional dref storing the maximal set of worlds satisfying the material in its scope.¹⁹ It then requires this set of worlds to be disjoint from the proposal dref.

- (63) $\llbracket \text{NOT}_p^{p'}(\varphi) \rrbracket^{(g,h)} = \mathbb{T}$ iff
- a. $\llbracket \text{max}^{p'}(\varphi) \rrbracket^{(g,h)} = \mathbb{T}$ and
 - b. $h(p) \cap h(p') = \emptyset$
- (64) $\llbracket \text{max}^p(\varphi) \rrbracket^{(g,h)} = \mathbb{T}$ iff
- a. $\llbracket [p] \wedge \varphi \rrbracket^{(g,h)} = \mathbb{T}$ and
 - b. there is no h' s.t. $\llbracket [p] \wedge \varphi \rrbracket^{(g,h')} = \mathbb{T}$ and $h(p) \subsetneq h'(p)$

We immediately capture the projection facts of a sentence like (57) above using this treatment of negation: the appositive within the scope of

¹⁹Independent evidence that negation introduces a propositional dref is provided by modal subordination discourses like *Linus does not have a car. He would have nowhere to park it* (Brasoveanu 2010:486, (131)) or more simply, *John did not jump. He would have died*. In both cases, the modal *would* is anaphoric to the proposition negated in the previous sentence. Our analysis of negation – which builds on Stone & Hardt (1999) and Brasoveanu (2010) (p. 497, fn. 24 in particular) – immediately predicts the felicity of these discourses because negation is not a test and introduces a dref storing the worlds satisfying the material in its scope. We would like to thank Philippe Schlenker for his comments about this issue.

negation is not problematic because the appositive always tests p^{cs} , ignoring the propositional dref introduced by negation. The result is that the logical forms in (65a) and (65b) below are equivalent.

- (65) a. John did not $_{p'}^{p'}$ [see $_{p'}$ Bill [who was hiding $_{p^{cs}}$]appos] $_{neg}$
 b. John did not $_{p'}^{p'}$ [see $_{p'}$ Bill] $_{neg}$ [who was hiding $_{p^{cs}}$]appos

The translation in (66), which is depicted in (67), shows how we derive the correct interpretation even when negation scopes over the appositive. For this example, assume that John saw Bill only in w_3 and that Bill was hiding only in w_1 and w_3 .

- (66) $[p] \wedge p \subseteq p^{cs} \wedge [x] \wedge x = \text{JOHN} \wedge [y] \wedge y = \text{BILL} \wedge$
 $\text{NOT}_{p'}^{p'} (\text{SEE}_{p'}(x, y) \wedge \text{HIDE}_{p^{cs}}(y)) \wedge$
 $[p^{cs}] \wedge p^{cs} = p$

(67)

$\{w_1, w_2, w_3\}$
$\{w_1, w_2\}$
$\{w_1, w_3\}$
$\{w_2, w_3\}$
$\{w_1\}$
$\{w_2\}$
$\{w_3\}$

$\xrightarrow[\text{1:AT-ISSUE}]{[p] \wedge p \subseteq p^{cs} \wedge [x] \wedge x = \text{JOHN} \wedge [y] \wedge y = \text{BILL}}$

p^{cs}	p	x	y
$\{w_1, w_2, w_3\}$	$\{w_1, w_2, w_3\}$	john	bill
$\{w_1, w_2, w_3\}$	$\{w_1, w_2\}$	john	bill
$\{w_1, w_2, w_3\}$	$\{w_1, w_3\}$	john	bill
	...		
$\{w_1, w_2\}$	$\{w_1, w_2\}$	john	bill
$\{w_1, w_2\}$	$\{w_1\}$	john	bill
$\{w_1, w_2\}$	$\{w_2\}$	john	bill
$\{w_1, w_3\}$	$\{w_1, w_3\}$	john	bill
$\{w_1, w_3\}$	$\{w_1\}$	john	bill
$\{w_1, w_3\}$	$\{w_3\}$	john	bill
$\{w_2, w_3\}$	$\{w_2, w_3\}$	john	bill
$\{w_2, w_3\}$	$\{w_2\}$	john	bill
$\{w_2, w_3\}$	$\{w_3\}$	john	bill
$\{w_1\}$	$\{w_1\}$	john	bill
$\{w_2\}$	$\{w_2\}$	john	bill
$\{w_3\}$	$\{w_3\}$	john	bill

$\xrightarrow[\text{2:NEG}]{\text{NOT}_{p'}^{p'} (\text{SEE}_{p'}(x, y) \wedge \dots)}$

p^{cs}	p	p'	x	y
$\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$	$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_3\}$	john	bill
	...			
$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_3\}$	john	bill
$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_1\}$	$\{\mathbf{w}_3\}$	john	bill
$\{\mathbf{w}_1, \mathbf{w}_2\}$	$\{\mathbf{w}_2\}$	$\{\mathbf{w}_3\}$	john	bill
$\{\mathbf{w}_1, \mathbf{w}_3\}$	$\{\mathbf{w}_1\}$	$\{\mathbf{w}_3\}$	john	bill
$\{\mathbf{w}_2, \mathbf{w}_3\}$	$\{\mathbf{w}_2\}$	$\{\mathbf{w}_3\}$	john	bill
$\{\mathbf{w}_1\}$	$\{\mathbf{w}_1\}$	$\{\mathbf{w}_3\}$	john	bill
$\{\mathbf{w}_2\}$	$\{\mathbf{w}_2\}$	$\{\mathbf{w}_3\}$	john	bill

$\xrightarrow[\text{3:APPOS}]{\dots \text{HIDE}_{p^{cs}}(y)}$

p^{cs}	p	p'	x	y
$\{\mathbf{w}_1, \mathbf{w}_3\}$	$\{\mathbf{w}_1\}$	$\{\mathbf{w}_3\}$	john	bill
$\{\mathbf{w}_1\}$	$\{\mathbf{w}_1\}$	$\{\mathbf{w}_3\}$	john	bill

$\xrightarrow[\text{4:AT-ISSUE}]{[p^{cs}] \wedge p^{cs} = p}$

p^{cs}	p	p'	x	y
$\{\mathbf{w}_1\}$	$\{\mathbf{w}_1\}$	$\{\mathbf{w}_3\}$	john	bill

We capture the intuitively correct truth conditions in (66) regardless of the appositive's scope because negation requires the maximal set of worlds stored in p' to be disjoint from p , but the appositive always directly updates p^{cs} . Thus, it can have no effect on the proposition that the speaker asserts to be false.²⁰

Ignoring the final two conjuncts indicating acceptance, the formula in (66) above is true just in case there is a proposal p that is disjoint from the worlds in which John sees Bill and p is a subset of the p^{cs} , which are all worlds where Bill is hiding. The two assignments that remain after the third update in (67) are precisely those that meet these conditions.

As the analysis of negation shows, the core idea is to assign the normal truth conditions to the relevant operators, but to calculate them using the maximized propositional drefs that they introduce. Since appositives always target p^{cs} , they cannot interact with these operators, even if the appositive is in their scope. Beyond negation, the semantic clause in (68) below shows how we treat a modal like *might* (once again,

²⁰Note that while the appositive cannot alter the propositional dref p' introduced by negation, it can make the negated proposition true by rendering p^{cs} , and *a fortiori* its subset p , disjoint from the negation-contributed dref p' . Such examples are grammatical, as expected, but pragmatically odd because they render the proposal uninformative.

- (1) #John didn't see Bill, who wasn't seen by John.

we build on Stone 1999, Stone & Hardt 1999, Brasoveanu 2010).

- (68) $\llbracket \text{MIGHT}_p^{p'}(\varphi) \rrbracket^{(g,h)} = \mathbb{T}$ iff
- $\llbracket \mathbf{max}^{p'}(\varphi) \rrbracket^{(g,h)} = \mathbb{T}$ and
 - for all $\mathbf{w} \in h(p)$, $\mathbf{MB}(\mathbf{w}) \cap h(p') \neq \emptyset$

The modal stores in p' the maximal set of worlds satisfying φ . It then checks that this set has a non-empty intersection with the modal base \mathbf{MB} that the modal verb *might* is associated with at each world in the proposal p . Just as in the case of negation, an appositive in the scope of *might* will necessarily update p^{cs} and not the propositional dref p' introduced by the modal. This correctly predicts that the appositive will have no effect on the modal claim.

- (69) John might $_p^{p'}$ [fight $_{p'}$ Bill [who is a professional boxer $_{p^{cs}}$] $_{appos}$] $_{might}$
- (70) a. **New proposal:** $[p] \wedge p \subseteq p^{cs} \wedge$
b. **Issue:** $[x] \wedge x = \text{JOHN} \wedge [y] \wedge y = \text{BILL} \wedge$
c. **Modal (issue ctd.):** $\text{MIGHT}_p^{p'}(\text{FIGHT}_{p'}(x, y) \wedge \dots$
d. **Appositive:** $\dots \text{PRO-BOXER}_{p^{cs}}(y)) \wedge$
e. **Acceptance:** $[p^{cs}] \wedge p^{cs} = p$

The proposal in (70) is true just in case p' stores the maximal set of worlds in which John fights Bill and this set has a non-empty intersection with the modal base \mathbf{MB} associated with the modal verb *might* at each world in the proposal p (which is a subset of the Context Set p^{cs}). Moreover, the Context Set p^{cs} is required to contain only worlds in which Bill is a professional boxer. As in the case of negation, the appositive content is not part of the modal claim but projects through to the Context Set, even though the appositive is in the scope of the modal verb *might*.

In the interest of brevity, we will not provide extensions of the analysis to other operators such as attitude predicates and conditional antecedents. We hope that it is clear that a similar strategy is feasible and that appositive content will always project past these elements for similar reasons.²¹

²¹For example, to account for conditionals we can just treat the antecedent/*if*-clause as introducing a modal dref that is subsequently picked up by the consequent/matrix clause, along the lines of Stone (1999) and Brasoveanu (2010). For attitude predicates like *believe*, we could let the attitude introduce a new propositional dref storing the maximal set of worlds satisfying the embedded clause. The attitude verb would further require this proposition to be a superset of the worlds that conform to the attitude holder's beliefs at each world in the proposal p . Appositives will be unable to affect the content of a conditional antecedent or propositional object of the attitude verb since they would update p^{cs} directly.

Instead, we turn to a related puzzle about the scope of appositive anchors, whose solution follows naturally under an account in which appositives directly update the Context Set.

5.1 The scope of appositive anchors

Our approach allows for appositive updates to project without interfering with the truth conditions of expressions in which they are embedded. That being said, while the appositive itself does not have a direct scopal interaction with such operators, the presence of an appositive does affect the scopal possibilities of its anchor DP, i.e., the DP it modifies.

For instance, the indefinite in (71) below can have both wide and narrow scope relative to negation. But the indefinite that the appositive in (72) is anchored to has only the wide scope reading: there must be a particular book that Mary recommended which John didn't read. Examples (73-74) show similar facts for the interaction of indefinites and modals.

- (71) John didn't read a book.
- (72) John didn't read a book, which Mary had recommended to him.
- (73) Mary must meet with a student.
- (74) Mary must meet with a student, who wants to join her lab.

We assimilate these data to the general fact that indefinites in the scope of negation and modals introduce drefs that are not available for cross-sentential anaphora. The explanation is more complicated than in the typical case of cross-sentential anaphora though, because appositives can be interpreted within the scope of negation and modals under our analysis.

This makes it impossible to explain the failure of narrow scope indefinite anchors by claiming that negation/modals contribute tests, and drefs introduced in their scope are not available for anaphora from outside their scope: we allow appositives to be syntactically in the scope of negation/modals, so any drefs introduced in the scope of these operators should be available for anaphora from within the appositives.²²

²²Note that in fact, we need to make the changes we are about to propose anyway if we want to account for the failure of cross-sentential anaphora to indefinites in the scope of operators like negation. The reason is that negation, modals, etc. are not tests under our analysis (see clauses (63) and (68)), so we predict that drefs introduced in their scope should be available to anaphora from outside their scope.

The idea behind our proposal²³ is that drefs introduced by indefinites will only be defined relative to a particular subset of possible worlds. If lexical predicates presuppose that their arguments are defined throughout the set of worlds relative to which they are interpreted, then the appositive will not be able to target an anchor interpreted in the scope of an operator like negation. In these cases, the appositive will presuppose that the dref introduced by its anchor is defined throughout p^{cs} , when in fact it is only defined throughout the propositional dref introduced by negation.

This will become clearer soon when we work through an example. But first we need to formally define what it means to take drefs for individuals and relativize them to possible worlds. The answer is basically the same as Montague’s: we treat them as (partial) individual concepts. That is, for any assignment g and variable x intended to be a dref for individuals, $g(x)$ is a partial function from a non-empty subset of the set of worlds \mathfrak{W} to the set of individuals \mathfrak{D} . Introducing new individual drefs is therefore always relativized to a propositional dref, as shown in (75) below.

- (75) $\llbracket [x_p] \rrbracket^{(g,h)} = \mathbb{T}$ iff
- a. for any variable v (of any type) s.t. $v \neq x$, we have that $g(v) = h(v)$, and
 - b. $\begin{cases} \mathbf{Dom}(h(x)) = h(p^{cs}) & \text{if } p \subseteq p^{cs} \text{ is the at-issue proposal} \\ \mathbf{Dom}(h(x)) = h(p) & \text{otherwise} \end{cases}$

The definition in (75) says that h differs from g at most with respect to the partial individual concept x , which is defined only for the worlds in the propositional dref p that x is relativized to, i.e., formally, the domain $\mathbf{Dom}(h(x))$ of the partial function assigned to the individual concept x is the set of worlds $h(p)$ currently assigned to p .

If p is the at-issue proposal dref, we require the individual concept x to be defined not only for the worlds in p , but for all the worlds in the Context Set p^{cs} , which is a superset of p . This is a slight technicality needed to allow for anaphora from an appositive clause to an indefinite in the main clause, e.g., *A^x man and a woman, who seemed to be his_x sister, were walking silently down the corridor.*

But to understand the main import of (75) above, we can ignore this technicality (we will come back to it) and focus on the simpler “otherwise” condition. Consider, for example, the two-sentence discourse in (76) below. Anaphora from the second sentence to the individual dref y introduced in the first sentence under negation is infelicitous.

²³Building on Stone (1999) and Brasoveanu (2010). See the discussion of *de re* vs. *de dicto* readings for indefinites under modals in Brasoveanu (2010: 498-499), and discourse (172) on p. 499 in particular.

(76) John^x isn't^{p'} eating a^y sundae. #It_y is melting fast.

We analyze this infelicity as a presupposition failure: predicates like MELT-FAST_p(*y*) in the second sentence presuppose that their individual arguments – *y* in this case – are defined/exist in every world of the propositional dref *p* relative to which they are evaluated. In general:

- (77) Lexical relations – final version with existence presuppositions:
- $\llbracket R_p(x_1, \dots, x_n) \rrbracket^{\langle g, h \rangle}$ presupposes that for any $i \in \{1, \dots, n\}$, $h(p) \subseteq \mathbf{Dom}(h(x_i))$.
 - If its presuppositions are satisfied, $\llbracket R_p(x_1, \dots, x_n) \rrbracket^{\langle g, h \rangle} = \mathbb{T}$ iff $g = h$ and for all worlds $\mathbf{w} \in h(p)$, $\langle h(x_1)(\mathbf{w}), \dots, h(x_n)(\mathbf{w}) \rangle \in \mathcal{I}_{\mathbf{w}}(R)$

But the propositional dref *p* in the second sentence of discourse (76) above is a subset of the Context Set dref p^{cs} , which in turn has already been constrained by the first sentence to be disjoint from the propositional dref p' introduced by negation.

Thus, given that the individual dref *y* is introduced in the scope of negation, it is relativized to the negation dref p' and is therefore defined only for the worlds in p' . Hence it cannot be defined for any of the worlds in *p*, which are disjoint from the p' -worlds.

For concreteness, we provide the semantic representation of discourse (76) in (78) below.

- (78) a. **Issue (1st sentence):** $[p_1] \wedge p_1 \subseteq p^{cs} \wedge [x_{p_1}] \wedge x = \text{JOHN} \wedge$
 b. **Negation (issue ctd.):** $\text{NOT}_{p_1}^{p'}([y_{p'}] \wedge \text{SUNDAE}_{p'}(y) \wedge$
 $\text{EAT}_{p'}(x, y)) \wedge$
 c. **Acceptance:** $[p^{cs}] \wedge p^{cs} = p_1 \wedge$
 d. **New issue (2nd sentence):** $[p_2] \wedge p_2 \subseteq p^{cs} \wedge$
 e. **Presupposition failure (new issue ctd.):**
 MELT-FAST_{p₂}(*y*) presupposes that $h(p_2) \subseteq \mathbf{Dom}(h(y))$,
 but this is not satisfied because $h(p_2) \subseteq h(p^{cs})$ and
 we already know that $h(p^{cs}) \cap \mathbf{Dom}(h(y)) = \emptyset$
 because $\mathbf{Dom}(h(y)) = h(p')$

Thus, relativizing individual drefs to propositional drefs as in definition (75) above is crucial to deriving the infelicity of anaphora in (76).

We can account for the infelicitous anaphora in (79) below (based on examples from Stone 1999) in a parallel way. The individual dref *y* introduced in the scope of the modal verb *might* is defined only for the worlds of the epistemic possibility p' brought up by the modal. But since not all the at-issue proposal worlds in the second sentence are

guaranteed to also be p' -worlds, we once again have a presupposition failure.

(79) John^x might^{p'} be eating a^y sundae. #It_y is melting fast.

The story of anaphora across modal environments is however not that simple (which will bring us back to the p^{cs} -related technicality in definition (75) above). Consider the felicitous example in (80) below and compare it to the infelicitous example in (79) above: (80) shows that anaphora from within the scope of a modal to an individual dref that is only known to exist in the Context Set p^{cs} is felicitous.

(80) John^x has a^y sundae. He_x might^{p'} share it_y.

We can account for the asymmetry between (79) and (80) if we add some general rules of default inference across modal environments. A fairly uncontroversial one (see for example Geurts 1999 and references therein) is that individuals that are assumed to exist in the actual world – or, in our terms, throughout the Context Set – can by default be assumed to also exist in non-factual modal environments, unless their existence in such environments is explicitly denied (this is exactly the type of inference we use to evaluate counterfactual conditionals, for example).

We can then take the p^{cs} -related technicality in definition (75) above to reify a similar default inference: given that the default conversational future (to use the terminology in Farkas & Bruce 2010) for the at-issue proposal p is to be accepted as the new Context Set p^{cs} , we feel justified in assuming that whatever individuals are brought to salience in the proposal can be taken to exist throughout the Context Set.

Against this backdrop, which is independently needed to account for anaphora across modal environments, the interaction between the presence of appositives and the scopal properties of their indefinite anchors follow automatically. Recall that what we want to account for is that appositives force their indefinite anchors to take wide scope over operators like negation.

(81) John^x didn't^{p'} read a^y book, which_y Mary^z had recommended to him_x.

Since the appositive clause updates p^{cs} directly, all lexical relations that make up the appositive update will presuppose that their arguments are defined throughout p^{cs} . In turn, these presuppositions can be met only if indefinites like *a^y book* in (81) above take wide scope.

Assume the indefinite *a^y book* in (81) were to take narrow scope under negation. The translation in (82) below shows that if the dref y

contributed by the indefinite is introduced in the scope of negation, it cannot satisfy the presupposition contributed by the appositive update $\text{RECOM}_{p^{cs}}(z, y, x)$ (i.e., z recommended y to x) that $h(p^{cs}) \subseteq h(y)$. Since y exists only throughout the negation worlds p' , it cannot also exist throughout the set of worlds p^{cs} if the proposal p were to be accepted, and that's because p and p' are required to be disjoint by the negation operator itself.

- (82) a. **Issue:** $[p] \wedge p \subseteq p^{cs} \wedge [x_p] \wedge x = \text{JOHN} \wedge$
 b. **Negation (issue ctd.):** $\text{NOT}_{p'}^{p'}([y_{p'}] \wedge \text{BOOK}_{p'}(y) \wedge$
 $\text{READ}_{p'}(x, y) \wedge \dots$
 c. **Appositive:** $\dots [z_{p^{cs}}] \wedge z = \text{MARY} \wedge$
 $\text{RECOM}_{p^{cs}}(z, y, x)) \wedge$
 d. **Acceptance:** $[p^{cs}] \wedge p^{cs} = p$

Thus, if the indefinite anchor takes narrow scope relative to negation, there is no way to both satisfy the existence presupposition contributed by the appositive update and at the same time, accept the at-issue proposal.

But when the indefinite takes wide scope, as shown in (83) below, there is no such problem: all the existence presuppositions can be satisfied and at the same time, the proposal can be accepted.

- (83) a. **Issue:** $[p] \wedge p \subseteq p^{cs} \wedge [x_p] \wedge x = \text{JOHN} \wedge$
 b. **Issue (ctd.):** $[y_p] \wedge \text{BOOK}_p(y) \wedge$
 c. **Negation (issue ctd.):** $\text{NOT}_{p'}^{p'}(\text{READ}_{p'}(x, y) \wedge \dots$
 d. **Appositive:** $\dots [z_{p^{cs}}] \wedge z = \text{MARY} \wedge$
 $\text{RECOM}_{p^{cs}}(z, y, x)) \wedge$
 e. **Acceptance:** $[p^{cs}] \wedge p^{cs} = p$

In sum, we see that our account correctly predicts obligatory wide scope for the indefinite anchors of appositives without forcing the appositive to take wide scope in the syntax. Instead, the analysis is parallel to the independently-needed account of anaphora across modal environments.

5.2 Some non-projecting appositives

Thus far, we have focused exclusively on appositive relative clauses, and provided an account of their projection past propositional operators. In this subsection, we briefly turn to nominal appositives and provide some preliminary thoughts on how the account may be extended to capture them.

In particular, while we can explain the scopal inertness of relative appositives, it has been proposed that certain types of nominal appositives can take narrow scope. For example, Wang et al. (2005) present the following examples where the appositive content can be moved into the issue while preserving paraphraseability.

- (84) a. If a professor, a famous one, publishes a book, he will make a lot of money.
b. If a professor publishes a book and he is famous, he will make a lot of money.
- (85) a. John believes that a professor, a quite famous one, published a new book.
b. John believes that a quite famous professor published a new book.

In (84), the indefinite can scope under the conditional antecedent regardless of whether it acts as an appositive anchor, while (85) shows that some indefinite appositive anchors seem to have *de dicto* readings.

These examples are potentially problematic for a theory in which appositives always impose their updates on the context set – assuming that nominal appositives are derived from their relative counterparts, as Doron (1994) and del Gobbo (2003) propose. As we have seen, directly updating p^{cs} guarantees widest scope in the system developed here, which is not an available/the only available reading for (84-85).

We will however show that nominal appositives do not behave like relative appositives in other ways – most importantly, they can interact with the QUD. This suggests that in these cases, they are appositive in prosody only. Our preliminary analysis is that the appositives in (84-85) are not appositives *per se* but *corrections* that target the proposal. As such, they differ both in their discourse behavior and their scopal properties, a connection which our account predicts.

First, note that not all nominal appositives exhibit the low-scoping behavior. It seems to be most readily available if the appositive has a matching quantificational head and contains *one*-anaphora. But the presence of *one*-anaphora is not the only feature distinguishing this type of appositive. In fact, as long as the quantifiers match, we find nominal appositives anchored to strong quantifiers in this construction, which is not always the case for relative-clause appositives, as the contrasts below show:

- (86) a. The dean will be happy if every professor, every famous one, publishes a book next year.
b. *The dean will be happy if every professor, every famous linguist, publishes a book next year.

- c. *The dean will be happy if every professor, who are famous linguists/who is a famous linguist, publishes a book next year.
- (87)
- a. If no professor, no boring one, comes to the party, it will be good.
 - b. *If no professor, no boring linguist, comes to the party, it will be good.
 - c. *If no professor, who are boring linguists/who is a boring linguist, comes to the party, it will be good.²⁴

These facts show that the appositives discussed in Wang et al. (2005) are a morpho-syntactically distinct subtype of nominal appositive, which we will call *one*-asides.

Importantly, in addition to having different constraints on the type of DPs they can get anchored to, *one*-asides are also semantically distinct from other relative or nominal appositives. In particular, not only are they able to take narrow scope, they are also able to interact with the QUD in ways other appositives cannot even if they are nominal.

For example, as we have seen, relative-clause appositive content cannot be challenged with polarity-particle answers. While this also holds for most nominal appositives, as shown in (88) below, the content of *one*-asides behaves as if it is part of the issue.

- (88)
- a. A: If John, a famous professor, writes a book, he will make a lot of money.
 - b. B: No way! (His book could flop // #He's not famous.)
- (89)
- a. A: If a professor, a famous one, publishes a book, he will make a lot of money.
 - b. B: No! (#Some professors publish books that flop // Some famous professors publish books that flop.)

Example (89) above shows that the bare particle answer must target not just the content of the appositive anchor, but also that which is carried by the appositive, which provides a clear contrast with normal nominal appositives. Since we have argued (following Farkas & Bruce 2010) that bare particle answers respond to the proposal, this means that *one*-asides must update the proposal p and not the Context Set p^{CS} directly. It is not surprising then that they can take narrow scope since under our analysis, scopal inertness arises from targeting p^{CS} .

²⁴Interestingly, if the anchor noun is plural, relative-appositive constructions improve, e.g., *The dean will be happy if all professors, who are (all) famous linguists, publish a book next year*, or *If no professors, who are (all) boring linguists, come to the party, it will be good*.

While we leave a more detailed investigation of *one*-asides and other nominal appositives for future work, we hope to have shown that they are not counterexamples to the present analysis. In fact, they provide additional support for it since even in these exceptional *one*-aside cases, the facts from projection and the failure to interact with the QUD never diverge, as our account predicts.²⁵ Our basic idea (which we leave unformalized here) is that *one*-asides are corrections to the proposal, which accords with intuition: examples like (84-85) feel like corrected assertions. If the speaker was to make the assertion all over again, s/he would have most probably used a paraphrase without the *one*-aside.

5.3 Summary of the proposed projection account

In sum, a major challenge for incremental, unidimensional treatments of appositives is to provide an account that derives projection past at-issue operators from the their special discourse status. While difficulties in interpreting them in the scope of such operators can conceivably be met by forcing the appositives to adjoin high in the syntax, such an account is less parsimonious because it treats appositives as being exceptional in both their syntax and their semantics, rather than in the semantics alone.

The account developed here meets this challenge by treating at-issue operators like negation and modals as propositional indefinites that, just like appositives, toggle between the proposal dref and the propositional dref they introduce. The wide-scope behavior of appositives follows from the fact that they (almost) always target the Context Set dref p^{cs} directly.

Our account also explains why appositives with indefinite anchors force those indefinites to take wide scope. Once again, we do not make recourse to syntax but instead show that these facts follow from the requirement that indefinites must take wide scope over modal operators in order to be available for particular kinds of cross-sentential anaphora.

²⁵Within the framework introduced here, *one*-asides can be analyzed as either targeting the old proposal or introduce a new proposal that strengthens the old one. The choice between these two possible analyses is partly determined by how we want to analyze comma-intonation. If comma-intonation is always associated with a shift in the dref that indexes lexical predicates, *one*-asides should probably be analyzed as propositional indefinites that introduce a new proposal which strengthens/corrects the old one.

6 Conclusion and future directions

In this paper, we have presented a dynamic semantics account of at-issue and appositive content that captures the senses in which the two are truly separate, while accounting for the systematic anaphoric phenomena that cross the boundary between these two kinds of content.

The core of the account is the idea that appositive content differs from at-issue content principally at the level of discourse negotiation. The analysis formalizes this intuition by proposing a dynamic semantics where the two kinds of content update the Context Set in two different ways. Appositives *impose* their content on the Context Set directly, while at-issue content introduces a new discourse referent and *proposes* that its contents be added, subject to ratification by the addressee. Crucially, since this distinction can be made within a semantics that is *incremental* and *unidimensional*, the resulting account is able to successfully capture the fact that we can robustly establish anaphoric connections between at-issue and appositive content.

Finally, we have seen that the core analysis can be extended to allow for a semantic account of another sense in which the content of appositives is “separate” from at-issue content: its projection past operators. Whereas previous unidimensional accounts of appositives predominantly give syntactic treatments of such facts, we have presented a semantics that derives projection from the difference in update procedure itself without the need for additional syntactic assumptions.

While our analysis can account for the behavior of canonical appositives, the primary area for future work is to explore the interpretation and distribution of non-canonical appositives – expressions with appositive morphology and prosody, but which interact with the context in novel ways. We have already shown how our analysis predicts that *one-asides*, which can scope under various operators, can also interact with the QUD. Another issue to investigate is the contrast between medial and final appositives.

In addition to the data provided in §3.2 involving responses and tags, there is also an asymmetry between medial and final appositives regarding their ability to host presupposition triggers. While presuppositions in clause-final appositives can be easily satisfied by at-issue material, parallel examples with clause-medial appositives are quite difficult to construct:

(90) John kissed Mary, who kissed HIM TOO.

(91) ??John kissed Mary, who kissed HIM TOO, at the concert in the park.

A further set of facts that demonstrate an asymmetry between medial

and final appositives is the interpretation of the temporal adverb *then*. In clause-final appositives like (92) below, *then* most readily indicates that the event in the appositive took place at a later time than the event in the main clause (a rough paraphrase being *subsequently*). In clause-medial appositives like (93), however, *then* indicates that the event described in the appositive took place at the same time as the at-issue event, a rough paraphrase being *at that time*.

- (92) Every springtime they migrate out of the sea and swim upriver to reproduce by giving birth to live young, who **then** spend an undetermined amount of time living on land as what we call zucchini slugs. (COCA)
- (93) In fact, while she was filming *Fresh Prince*, her mother, who **then** worked as a librarian, would call her with Black history stories. (COCA)

Taken together, these observations demonstrate that despite having the same surface form (at least in English), there is a persistent asymmetry between clause-final and clause-medial appositives – an idea previously suggested by [del Gobbo \(2003\)](#), based on quite different data (the relative felicity of clause-final appositives anchored on quantifiers like *many* and *most*).

While it's not immediately clear how [del Gobbo \(2003\)](#)'s data fit into this picture, the data from responses, tags, presupposition, and *then* seem to suggest that final appositives often behave more like conjunctions or separate sentences rather than true appositives.²⁶ We can cash out this idea in our formalism if final appositives are ambiguous between targeting p^{cs} (as we have analyzed them up until now) or introducing their own proposal and thereby forcing an intermediate acceptance of the old proposal.

That is, a final appositive like (94) below can have the translation in (94a) or (94b).

- (94) He took care of his husband, who had prostate cancer.
- a. **Proposal:** $[p] \wedge p \subseteq p^{cs} \wedge$
Issue: $\text{TAKE-CARE}_p(x, y) \wedge \text{HUSBAND}_p(y, x) \wedge$
Appositive: $\text{HAVE-CANCER}_{p^{cs}}(y) \wedge$
Acceptance: $[p^{cs}] \wedge p^{cs} = p$
- b. **Proposal:** $[p] \wedge p \subseteq p^{cs} \wedge$
Issue: $\text{TAKE-CARE}_p(x, y) \wedge \text{HUSBAND}_p(y, x) \wedge$
Acceptance: $[p^{cs}] \wedge p^{cs} = p \wedge$
Appositive: $[p'] \wedge p' \subseteq p^{cs} \wedge \text{HAVE-CANCER}_{p'}(y)$

²⁶We want to thank Tamina Stephenson for discussion of this issue.

This is basically the account of medial vs. final appositives in [Koev \(2012\)](#).²⁷

All the differences we have identified between medial and final appositives can be explained if only the latter can have a representation like the one in (94b).

First, the analysis explains why final appositives can answer questions or raise issues, as evidenced by tag questions. We have argued that interacting with the QUD is a property of proposals, so if final appositives act like separate sentences by introducing their own proposal, we predict that they should interact with the QUD.

The analysis also predicts the felicity of satisfying appositive-internal presuppositions relative to the at-issue proposal when the appositive is final, but not medial. Recall that presuppositions are tests on the current p^{cs} . If final appositives can introduce their own proposal, thereby forcing the acceptance of the previous proposal, then the p^{cs} will reflect the conditions imposed before the appositive, which can license the appositive's presuppositions.

Finally, we account for the interpretation of *then* in final appositives under the assumption that only proposals can move the topic time forward.

The analysis sketched here, of course, leaves unexplained *why* it is only final appositives that can make new proposals. One possibility is to follow previous authors, for example [del Gobbo \(2003\)](#), who have argued for a syntactic operation of restructuring that can adjoin syntactic material to a text node in a discourse-level syntax as long as the operation does not change the linear order of terminals. Under the natural assumption that introducing a new proposal is a semantic property of constituents dominated by a text node, then the final/medial appositive asymmetries can be explained in an analysis like ours combined with restructuring.

A related idea, suggested to us by Nicholas Asher (p.c.), is that final appositives have a wider range of interpretations because they can enter into matrix-level discourse relations in a discourse structure (e.g., [Asher & Lascarides 2003](#)). Medial appositives would be subject to more constraints on their interpretation because they would be discourse-subordinate to the clause they are syntactically attached to, while final appositives can but need not be discourse subordinate in the same way. For us, matrix level discourse relations would involve introducing a new proposal and interacting with the QUD. In fact, the proposal put forth in [Koev \(2012\)](#) seems to be along these very lines.

²⁷As Koev, who builds on the early version of this paper ([AnderBois et al \(2010\)](#)), discusses in detail, this basic account of medial vs. final appositives is not yet exactly right. But this sketch is sufficient for the present purposes.

We leave a more detailed investigation of sentence-final appositives, as well as a more detailed comparison of relative appositives and various not-at-issue constructions documented in the literature (nominal appositives, expressives, speaker-oriented adverbials, slifting, evidentials etc.) for future work. However, we hope to have shown that our account of appositive relative clauses and the way we have formally fleshed out the proposal nature of at-issue content are useful steps towards this larger goal.

References

- Abusch, D. (2010). Presupposition Triggering from Alternatives. *Journal of Semantics* 27, 37-80.
- Amaral, P., C. Roberts & E.A. Smith (2007). Review of "The Logic of Conventional Implicatures". *Linguistics and Philosophy* 30, 707-749.
- AnderBois, S. (2010). Sluicing as Anaphora to Issues. *Proceedings of Semantics and Linguistic Theory* 20, N. Li & D. Lutz (eds.), eLanguage, 451-470.
- AnderBois, S., A. Brasoveanu & R. Henderson (2010). Crossing the Appositive/At-Issue Meaning Boundary. *Proceedings of Semantics and Linguistic Theory* 20, N. Li & D. Lutz (eds.), eLanguage, 328-346.
- Asher, N. & Lascarides, A. (2003). *Logics of conversation*. Cambridge University Press.
- Brasoveanu, A. (2010). Decomposing Modal Quantification. *Journal of Semantics* 27, 437-527.
- Brasoveanu, A. (2013). Modified Numerals as Post-Suppositions. *Journal of Semantics* 30, 155-209.
- Büring, D. (2003). On D-trees, Beans and B-accent. *Linguistics and Philosophy* 26, 511-45.
- Clark, H. & E. Schaefer (1989). Contributing to discourse. *Cognitive Science* 13, 259-294.
- Davies, Mark (2008-). The Corpus of Contemporary American English (COCA): 400+ million words, 1990-present. Available online at <http://www.americancorpus.org>.
- Demirdache, H. 1991. Resumptive chains in restrictive relatives, appositives and dislocation structures. Doctoral dissertation, MIT, Cambridge, MA.
- Doron, E. (1994). The discourse function of appositives. In the *Proceedings of IATL* 9.
- Farkas, D. & K. Bruce (2010). On Reacting to Assertions and Polar Questions. *Journal of Semantics* 27, 81-118.
- Geurts, B. (1999). *Presuppositions and Pronouns*, Amsterdam, Elsevier.

- Revised version of Geurts, B. 1995, *Presupposing*, PhD dissertation, University of Stuttgart.
- Ginzburg, J. (1996). Dynamics and the Semantics of Dialogue. *Language, Logic and Computation*. Vol. 1, J. Seligman & Dag Westerståhl (eds.), CSLI, 221-237.
- del Gobbo, F. (2003). *Appositives at the Interface*. PhD Thesis, Univ. of California, Irvine.
- Groenendijk, J. & M. Stokhof (1991). Dynamic Predicate Logic, in *Linguistics and Philosophy* 14, 39-100.
- Gunlogson, C. (2001). *True to Form: Rising and Falling Declaratives as Questions in English*, PhD dissertation, UC Santa Cruz.
- Hankamer, J. & I. Sag (1976) Deep and Surface Anaphora. *Linguistic Inquiry* 7, 391-428.
- Hardt, D. (1999) Dynamic Interpretation of VPE. *Linguistics and Philosophy* 22, 187-221.
- Harris, J. & C. Potts (2009). Perspective-shifting with appositives and expressives. *Linguistics and Philosophy* 32, 523-552.
- Heim, I. (1982). *The Semantics of Definite and Indefinite Noun Phrases*, PhD dissertation, University of Massachusetts, Amherst, published in 1988 by Garland, New York.
- Kamp, H. (2001). The Importance of Presupposition. *Linguistic Form and its Computation. Selected papers from the SFB 340*, C. Rohrer & A. Roßdeutscher (eds.), CSLI, 207-254.
- Koev, T. (2012). On the Information Status of Appositive Relative Clauses. *LNCS 7218 (LNAI): Amsterdam Colloquium 2011*, Springer, 401-410.
- Krahmer, E. (1998). *Presupposition and Anaphora*. CSLI.
- Merchant, J. (2001). *The Syntax of Silence*. OUP.
- Murray, S. (2009a). A Hamblin Semantics for Evidentials. *Proceedings of SALT XIX*.
- Murray, S. (2009b). Evidentials and Questions in Cheyenne. *Proceedings of SULA 5: Semantics of Under-represented Languages in the Americas*.
- Nouwen, R. (2007). On Appositives and Dynamic Binding, *Res. Lang. Comput.* 5, 87-102.
- Nouwen, R. (2010). A note on the projection of nominal appositives, (ms).
- Potts, C. (2005). *The Logic of Conventional Implicatures*, OUP.
- Potts, C. (2009). The dynamics of Apposition. Handout available at <http://www.stanford.edu/~cgpotts/talks/potts-stanford09-dynapp.pdf> (last checked: May 16, 2013).
- Roberts, C. (1996). Information Structure: Towards an integrated theory of formal pragmatics. *OSU Working Papers in Linguistics* 48, J. Yoon & A. Kathol (eds.), 91-136.

- van der Sandt, R. (1992). Presupposition Projection as Anaphora Resolution. *Journal of Semantics* 9, 333-377.
- Schlenker, P. (2009). Supplements within a Unidimensional Semantics I: Scope. *Proceedings of the Amsterdam Colloquium 2009*, M. Aloni & K. Schulz (eds).
- Schlenker, P. (2009). Supplements within a Unidimensional Semantics II: Epistemic Status and Projection. *Proceedings of NELS 2009*.
- Simons, M. & Tonhauser, J. & Beaver, D. & Roberts, C. (2010). What projects and why. *Proceedings of SALT 20*, 309-327. CLC Publications.
- Stalnaker, R. (1978). Assertion, in *Syntax and Semantics* 9, 315-332.
- Stone, M. & D. Hardt (1999). Dynamic Discourse Referents for Tense and Modals. *Proceedings of IWCS 2*, 287-299.
- Stone, M. (1999). Reference to Possible Worlds. RuCCS Report 49, Rutgers University.
- Wang, L. & B. Reese & E. McCready (2005). The projection problem of nominal appositives. *Snippets* 10, 287-299.

A The formal system

The final formal system:²⁸

- (1) Models: $\mathfrak{M} = \langle \mathfrak{D}, \mathfrak{W}, \mathfrak{I} \rangle$, where the domain of individuals \mathfrak{D} and the domain of possible worlds \mathfrak{W} are disjoint, and the basic interpretation function \mathfrak{I} assigns a subset of \mathfrak{D}^n to any n -ary relation R relative to any world \mathbf{w} : $\mathfrak{I}_{\mathbf{w}}(R) \subseteq \mathfrak{D}^n$.
- (2) We implicitly assume that we can build higher-order domains as needed, e.g., a domain for partial individual concepts,²⁹ a domain for propositions, etc.
- (3) Variables over:
 - partial individual concepts: x, y, \dots
 - worlds: w, w', \dots
 - propositions/sets of worlds: p, p', p^{cs}, \dots
- (4) The usual inventory of non-logical constants:
 - individual constants: JOHN, \dots

²⁸See Brasoveanu (2013: 181 et seqq) for a closely related set of definitions independently motivated by a different range of semantic phenomena.

²⁹Partial individual concepts could be encoded in various ways in a system that countenances only total functions., e.g., as functions mapping the worlds in the domain of a partial individual concept to singleton sets of individuals and all the other worlds to the empty set of individuals.

- properties: WOMAN, ...
 - binary relations: VISIT, ...
 - etc.
- (5) The interpretation function has the form $\llbracket \cdot \rrbracket^{\mathfrak{M}, \langle g, h \rangle}$, i.e., formulas denote binary relations between an input assignment g and an output assignment h . We usually omit the model superscript \mathfrak{M} .
- (6) Atomic formulas: lexical relations – preliminary version without existence presuppositions.
- a. $\llbracket x = y \rrbracket^{\langle g, h \rangle} = \mathbb{T}$ iff $g = h$ and $h(x) = h(y)$
 - b. $\llbracket x = \text{JOHN} \rrbracket^{\langle g, h \rangle} = \mathbb{T}$ iff $g = h$ and $\mathbf{Ran}(h(x)) = \{\mathcal{I}(\text{JOHN})\}$ ³⁰
 - c. $\llbracket p = p' \rrbracket^{\langle g, h \rangle} = \mathbb{T}$ iff $g = h$ and $h(p) = h(p')$
 - d. $\llbracket p \subseteq p' \rrbracket^{\langle g, h \rangle} = \mathbb{T}$ iff $g = h$ and $h(p) \subseteq h(p')$
 - e. $\llbracket \text{WOMAN}_p(x) \rrbracket^{\langle g, h \rangle} = \mathbb{T}$ iff $g = h$ and for all worlds $\mathbf{w} \in h(p)$, $h(x)(\mathbf{w}) \in \mathcal{I}_{\mathbf{w}}(\text{WOMAN})$
 - f. $\llbracket \text{VISIT}_p(x, y) \rrbracket^{\langle g, h \rangle} = \mathbb{T}$ iff $g = h$ and for all worlds $\mathbf{w} \in h(p)$, $\langle h(x)(\mathbf{w}), h(y)(\mathbf{w}) \rangle \in \mathcal{I}_{\mathbf{w}}(\text{VISIT})$
 - g. etc.
- (7) Atomic formulas: lexical relations – final version with existence presuppositions.
- a. $\llbracket R_p(x_1, \dots, x_n) \rrbracket^{\langle g, h \rangle}$ presupposes that for any $i \in \{1, \dots, n\}$, $h(p) \subseteq \mathbf{Dom}(h(x_i))$.
 - b. If its presuppositions are satisfied, $\llbracket R_p(x_1, \dots, x_n) \rrbracket^{\langle g, h \rangle} = \mathbb{T}$ iff $g = h$ and for all worlds $\mathbf{w} \in h(p)$, $\langle h(x_1)(\mathbf{w}), \dots, h(x_n)(\mathbf{w}) \rangle \in \mathcal{I}_{\mathbf{w}}(R)$
- (8) Atomic formulas: random assignment of values to variables.
- a. $\llbracket [w] \rrbracket^{\langle g, h \rangle} = \mathbb{T}$ iff for any variable v (of any type) s.t. $v \neq w$, we have that $g(v) = h(v)$
 - b. $\llbracket [p] \rrbracket^{\langle g, h \rangle} = \mathbb{T}$ iff for any variable v (of any type) s.t. $v \neq p$, we have that $g(v) = h(v)$
 - c. $\llbracket [x_p] \rrbracket^{\langle g, h \rangle} = \mathbb{T}$ iff
 - i. for any variable v (of any type) s.t. $v \neq x$, we have that $g(v) = h(v)$, and

³⁰We assume that any individual constant denotes the same entity in all possible worlds, so we do not explicitly relativize the interpretation of individual constants to worlds.

- ii. $\begin{cases} \mathbf{Dom}(h(x)) = h(p^{cs}) & \text{if } p \subseteq p^{cs} \text{ is the at-issue proposal} \\ \mathbf{Dom}(h(x)) = h(p) & \text{otherwise} \end{cases}$
- (9) Dynamic conjunction (interpreted as relation composition).
 $\llbracket \varphi \wedge \psi \rrbracket^{(g,h)} = \mathbb{T}$ iff
there exists a k such that $\llbracket \varphi \rrbracket^{(g,k)} = \mathbb{T}$ and $\llbracket \psi \rrbracket^{(k,h)} = \mathbb{T}$
- (10) Maximization.
 $\llbracket \mathbf{max}^p(\varphi) \rrbracket^{(g,h)} = \mathbb{T}$ iff
a. $\llbracket [p] \wedge \varphi \rrbracket^{(g,h)} = \mathbb{T}$ and
b. there is no h' s.t. $\llbracket [p] \wedge \varphi \rrbracket^{(g,h')} = \mathbb{T}$ and $h(p) \subsetneq h'(p)$
- (11) Negation.
 $\llbracket \mathbf{NOT}_p^{p'}(\varphi) \rrbracket^{(g,h)} = \mathbb{T}$ iff
a. $\llbracket \mathbf{max}^{p'}(\varphi) \rrbracket^{(g,h)} = \mathbb{T}$ and
b. $h(p) \cap h(p') = \emptyset$
- (12) Possibility modals.
 $\llbracket \mathbf{MIGHT}_p^{p'}(\varphi) \rrbracket^{(g,h)} = \mathbb{T}$ iff
a. $\llbracket \mathbf{max}^{p'}(\varphi) \rrbracket^{(g,h)} = \mathbb{T}$ and
b. for all $\mathbf{w} \in h(p)$, $\mathbf{MB}(\mathbf{w}) \cap h(p') \neq \emptyset$