

The morphosemantics of incremental plurality (Hualapai)*

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1 Introduction

An areal feature of the Indigenous languages of the Southwest United States and Northwest Mexico are morphologically complex systems of plur(action)ality.

- There are multiple instances of unrelated languages implementing so-called *incremental* morphology—that is, there is no one-to-one mapping between exponents and meanings (e.g., Seri (Baerman, 2016), Hualapai (Baerman, 2019), Salinan (Baerman, 2024), etc.).
- These languages have a list of plur(action)al meanings ordered by some notion of “more plural” (call it $<_s$ for semantic order), along with a list of exponents ordered by some morphosyntactically defined order (call it $<_m$ for morphological order).
- Paradigms are well-formed as long as these orders are in scale alignment. More precisely, $\alpha <_m \beta$ iff $[[\alpha]] <_s [[\beta]]$

Such systems immediately generate three questions:

1. How do we define the morphological order ($<_m$)?
 - It could be that some forms are longer than other forms, it could be that some forms are morphologically complex than other forms, it could be by fiat, etc.

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2. How do we define the semantic order ($<_s$)?

- It could be by entailment, it could be complexity of features (defined in various ways), etc.

3. What kind of theory of the morphosemantic interface could generate scale-based morphological systems?

The primary goal of this talk is to present a complete account, addressing all three questions for the Hualapai language.

2 Incremental plurality, a first look

We begin by looking at the case of Hualapai (Yuman), a language spoken in Northwest Arizona along the Colorado River.

- Watahomigie et al. (2001a) describes verbs as coming in one of four forms, with speakers describing higher forms as having a “more plural” interpretation.¹

- (1) a. **F1.** ɖagwan ‘one beats up someone’
b. **F2.** ɖaɖgwanj ‘two/a few beat up someone’
c. **F3.** ɖaɖgwan ‘many beat up someone’
d. **F4.** ɖaɖgwanj ‘many beat up many’

¹The situation is actually more complex, with verbs sometimes having more or less forms. For instance, this verb *dabil* also has a form *ɖabi:lj* for a plural (non-paucal) subject acting on a singular object, e.g., $\beta a:jach\ i' \text{ ɖabi:ljikwi}$ ‘People are burning a log’ (Watahomigie et al., 2001b, p. 299). These idea is that these four forms are paradigmatic in occurring across many verb stems.

- (2) a. **F1.** *dabil* ‘one burns one’
- b. **F2.** *ɖabilj* ‘two/a few burn one’
- c. **F3.** *ɖaɖbi:l* ‘one burns many’
- d. **F4.** *ɖaɖbi:lj* ‘many burn many’

There are some critical things to note:

1. The second forms involve a paucal subject reading, while the fourth forms involve a ‘greater plural’ subject and object reading, and the third forms involve either a greater plural subject or greater plural object depending on verb class.
2. Here we see the question of the semantic order ($<_s$) that we raised in the introduction—what semantic property could we ascribe to these forms so that we predict this order that they stand in? Note for instance, just to throw out an idea, note that it is not entailment.

The choice to discuss Hualapai plural inflection generally—in terms of first form, second form, etc.—rather than referring to particular affixes, has not been an accident.

- The reason is that the mapping from specific exponents to meanings is not deterministic.

Hualapai thus shows two features that we will also see when we consider plural verbal morphology in Seri.

1. There is no one-to-one mapping between meanings and their exponents.
2. Meanings, which have shown can be ordered, are instead assigned to exponents to form paradigms somewhat arbitrarily, but in a ways that respect this order.

This is what has been called *incremental* morphology (e.g., Baerman 2016), and is exemplified in the figure below.

- First, note that we cannot say that vowel length marks the second form because it actually marks the third form for stem *gilgyo* ‘tie something large’.
- Similarly, we cannot say that *-j* marks the second form because it actually marks third and fourth forms for stem *hwal* ‘dig’.
- This is what we mean by a lack of one-to-one mapping between exponents and meanings.

Form 1	Form 2	Form 3	Form 4	
<i>hwal</i>	<i>hwa:l</i>	<i>hwa:l-j</i>		‘dig’
<i>gilgyo</i>	<i>gilgyo-j</i>	<i>gilgyo:</i>	<i>gilgyo:-j</i>	‘tie’

At the same time, we see in this example an illustration of incrementality.

- Note that while vowel length marks different meanings in these two paradigms, in both cases having both vowel length and the *-j* affix marks a meaning further up the semantic scale we have established.

The result is that knowing that $\alpha <_m \alpha \oplus \beta$ means that while we may not know $\llbracket \alpha \rrbracket$ or $\llbracket \alpha \oplus \beta \rrbracket$, due to lack of one-to-one mapping, we do know that $\llbracket \alpha \rrbracket <_s \llbracket \alpha \oplus \beta \rrbracket$.

- To reiterate—the assignment of meanings to these exponents in this morphological order is in alignment with the semantic scale.

At least in this restricted case, there is an iconic logic to this pattern—doing $\alpha \oplus \beta$ must yield a more plural meaning than either doing α or β on its own.

- Could a system like this be easily treated via an appeal to iconicity?
- No!

The morphological objection

- It is not always the case that “more exponents” means “more plural”.
- Prefixal plural, like the *ji-* prefix, uniformly outrank other kinds of marking, i.e., vowel length, suffixation, or even, critically, both vowel length and suffixation.

Form 1	Form 2	Form 3	Form 4	
<i>jigyo</i>	<i>jigyo:-j</i>	<i>ji-jgyo</i>	<i>ji-jgyo:-j</i>	‘bite’

- The iconic logic falls apart here, which also would extend to other kinds of accounts—e.g., something along the lines of Manner. In what sense is prefixation more complex than suffixation such that it should yield more complex meanings?
- We don’t have a ton of time to look at paradigms in Seri, but I can assure we have the same kinds of patterns in Seri that make a simple iconic account fail on morphological grounds.

The semantic objection

Even if it were the case that an iconic account could work on morphological grounds, there would be semantic hurdles.

- The simplest kind of iconic account would say that that big forms involve big plurality.

- (3) a. There were people everywhere (sweeping gesture)
 b. There were people everywhere (bigger sweeping gesture)

In the case of Hualapai, it is not at all clear that higher forms require bigger pluralities.

- If one person burns 500 things and 20 people burn 20 things, well then one person burned many things (i.e., Form 3) and many people burned many things (Form 4), but it is not the case that we have higher cardinalities involved in the Form 4 scenario.
- Perhaps there is some iconic story you could tell about the meanings involved, but it must not be the simplest such account.
- Combined with the fact that the morphology is also not necessarily iconic, I am doubtful of this approach.

2.1 Interim conclusions

Where do we go from here:

- We need a real theory of how incremental systems work compositionally.
 - In the next section we will do this for Hualapai
 - In particular, we will argue that the exponents we see on the surface, while not one-to-one linked to truth conditions of verbs, are one-to-one linked to presuppositions which compositionally constrain possible meanings for verb-forms bearing those exponents.
 - Moreover, they constrain them in such a way that the patterns we see across paradigms emerge.

3 Incremental plurality in Hualapai

In the previous subsection, we have ruled out a null hypothesis. We now have to deal with the serious problems at hand:

- What should the truth conditions of the following forms be?
- How do we, based on those truth conditions, place those forms into a semantic order?

- How does the morphology we see in the surface contribute to the meanings of these forms, given the lack of one-to-one correspondence between exponents and meanings?
- How the morphology we see on the surface come to respect the semantic order given the lack of one-to-one correspondence between exponents and meanings?

- (4) a. **F1.** *dabil* ‘one burns one’
 b. **F2.** *ḁabilj* ‘two/a few burn one’
 c. **F3.** *ḁḁbi:l* ‘one burns many’
 d. **F4.** *ḁḁbi:lj* ‘many burn many’
- (5) a. **F1.** *ḁagwan* ‘one beats up someone’
 b. **F2.** *ḁḁḁgwanj* ‘two/a few beat up someone’
 c. **F3.** *ḁḁḁgwan* ‘many beat up someone’
 d. **F4.** *ḁḁḁgwanj* ‘many beat up many’

We will begin with fleshing out the meanings of these forms before turning to morphology and questions of composition.

Guiding Intuition: We should be thinking about the entire system of verbal plural marking in Hualapai in terms of vague notions of high/low cardinality.

- First, this is reflected in the translations given by Watahomigie et al. (2001b) throughout the text which are always of the form ‘*Two/A few X Y-ed*’ ‘*Many X Y-ed many Z*’, etc.
- Note that in no case do we have a simple singular vs. plural contrast—all “plurals” are translated with vague cardinality quantifiers: *two/a few, many, a lot*.

We should, of course, be skeptical of translations, but we see attested examples that are consistent with the meanings as described.

- Note that across the paradigms that there are no paucal object forms.
- This suggests that low cardinality objects should not require plural verb forms, which is correct.

Consider object number in the following example.

- The first conjunct makes it clear that there are two deer, but neither the verb *'u:k* “see” or *gae:* “shoot” bears third category plural forms, which should indicate object plurality.

(6) *Nya ḏáalach qwa:q nye:ḏik qwa:q hwak'm*
 nya ḏal-a-ch qwa:q nye:-ḏi-k qwa:q hwak-m
 I 1.father-Def-Subj deer 3/3.hunt-Temp/then-SS deer 3.be.two-DS
'u:k gae:kwiny.
 u:-k gae:-k-wi-ny
 3/3.see-SS 3/3.shoot-SS-Aux/do-Past

‘When he was hunting, my father saw two deer and shot them.’ (Watahomigie et al., 2001b, p. 335-6)

The following examples show that such forms indicating object plurality do exist for these verbs, which only emphasizes that non-atomic reference is not what is at issue for this species of “plural” marking.

(7) *bà'gweg'u:ja*
 ba'-gwe-g-'u:-j-a
 person-thing-Nom-see-PI-Agent
 ‘person who sees things/researcher’ (Watahomigie et al., 2001b, p. 202)

(8) *gáe:jthik jìjyámk vawímwiny*
 gae:-j-th-k ji-jiyam-k va-wim-wi-ny
 1/3.shoot-PI-really-SS 1/3.Pl-miss-SS 1/3.Emph-intensely.do-Aux/do-Past

‘I really shot at them but I missed them all.’ (Watahomigie et al., 2001b, p. 69)

Taking vague cardinality route has an additional virtue in that it connects with the fact, noted by Baerman (2019), that third and fourth category forms should be thought of in terms of pluractionality.

- In particular, with intransitives, the plural marking we can indicate repeated action.
- For instance, we have *ḏiv'ik* ‘one person kneels down once’ vs. *ḏiv'i:j'k* ‘one person kneels down multiple times’
- The generalization is that with intransitives, we can mark a paucal subject, but when we add more plural morphology, we get a clear multiple even reading.

- In the case of transitives, where there are more core arguments, these plural forms can implicate these extra arguments rather than the temporal trace, as we see for intransitives.

If third and fourth forms of the Hualapai verbal number paradigm involve distributive pluractionality, the requirement that the subject / object have a large, unspecified cardinality, would follow from the standard behavior of pluractionals.

- It is well known that pluractionality, while involving pluralities of events, only rarely demand mere non-atomicity (e.g., Henderson 2012, 2017; Lasersohn 1995; Wood 2007; Hofherr and Laca 2012).
- It is one of the ways in which pluractionality is not like the most common type of nominal plurality crosslinguistically.
- Instead, pluractional predicates are often only satisfied by plural events of a sufficiently large, though vague, cardinality.

In this case, third and fourth form transitives would involve distributive pluractionality.

- They would mandate there be a sufficiently large number of events (the plurality requirement),
- each of which having a distinct participant (the distributivity requirement),
- resulting in plurality of participants whose cardinality is large, though not directly specified.

Once again, there is evidence that we do, in fact, have a requirement for a distributive interpretation for these high cardinality third and fourth forms.

- Note that collective predicates like *ḏagáv'k/ḏigáv'k* ‘gather’ in Hualapai tolerate singular agreement in the clear presence of a plural subject.
- Note the lack of a distinction in plural agreement between gathering cattle in (9) and gathering people in (10).
- In both cases we have a singular subject verb despite the explicit plural marking in (10).

(9) *Waksích isavgó búkal ḏigáv'kyu.*
 waksi-ch isavgo buk(a)-l ḏigá-v-k-yu
 cow-SUBJ corral foot-at 3.GATHER-STATE-SS-AUX/be

‘The cattle gathered at the corner of the corral (or close to the fence of the corral).’ (Watahomigie et al., 2001b, p. 52)

- We saw this, for example, with *qwa:q hwak'm* ‘two deer’ in (6).
- We take plural morphology then to add constraints that actively force certain arguments to have particular kinds of plural interpretations.

We can see how this strategy works by considering the second category forms which are associated with a paucal interpretation of the subject.

- A virtue of our neo-Davidsonian account is that we can actually place constraints on the subject through constraints on the event argument, which is related to entities denoted by the subject through the appropriate theta-role function.
- Assuming CARD is a measure function, we can treat second form paucals in (14) as placing a vague, low, cardinality condition on the agent of the event as in (15).

(14) *Merich joq gwájik Bobm hwákak*
 Meri-ch joq gwajik Bob-m hwak-(a)k
 Mary-SUBJ juniper near Bob-COM 3.two.together-SS
gige:vkyu.
gigev-k-yu
 3.stand.PAUC-SS-AUX

‘Mary and Bob are standing beside the juniper tree.’ (Watahomigie et al., 2001b, p. 51)

This verb phrase in (14) can felicitously combine with the subject *Merich Bobm hwákak* ‘Mary and Bob’, because its cardinality is greater than one and less than the contextually specified standard for FEW.

(15) *gige:vkyu* $\rightsquigarrow \lambda x \lambda e [* \text{STAND}(e) \wedge \mathbf{ag}(e) = x$
 $\wedge \exists n [1 < n < \text{FEW}_{\text{STD}} \wedge \text{CARD}(\mathbf{ag}(e)) = n]]$

When we move to third category forms we have analytical options.

- For instance, we could provide a uniform semantics for such forms and let morphosyntactic considerations generate the difference between those that require a large cardinality subject and those that require a large cardinality object.
- In the absence of detailed syntactic work on Hualapai, we will instead focus on the truth conditions and treat the two subclasses of Hualapai transitive verbs as having the same semantic template, but targeting different thematic roles.
- Recall, though, that we want to treat third and fourth forms as pluractionals, in particular, as distributive pluractionals.

- The large cardinality of the subject/object is due to requiring a large cardinality of events be distributed across some argument.

To implement distributive pluractionality, we follow a long literature that notes that event pluralities must often be distinguished along some trace that provides the counting criterion (e.g., Henderson 2012; Lasersohn 1995; Pasquereau 2019 among others).

- To help with this notion, we will introduce a bit of notation to return the atomic subevents of an event that differ on trace γ .

(16) $\mathbf{E}_e(\gamma) = \{e' | e' \leq e \wedge \forall e'' \leq e' [\mathbf{atom}(e'') \rightarrow \gamma(e') = \gamma(e'')] \wedge \neg \exists e''' [\gamma(e''') = \gamma(e') \wedge e''' \not\leq e'] \}$
 ‘The set of events e' in e that share an image under γ ’

We can now handle forms like *ḏaḏbi:l* ‘one burns many’ in (17) which computes event pluralities with respect to the theme.

(17) *ḏaḏbi:l* $\rightsquigarrow \lambda y \lambda x \lambda e [* \text{BURN}(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y$
 $\wedge \exists m [\text{MANY}_{\text{STD}} < m \wedge \text{CARD}(\mathbf{E}_e(\mathbf{th})) = m]]$

We can also handle forms like *ḏaḏgwan* ‘many beat up someone’ in (17), which event pluralities determined via the agent.

(18) *ḏaḏgwan* $\rightsquigarrow \lambda y \lambda x \lambda e [* \text{BEAT}(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y$
 $\wedge \exists m [\text{MANY}_{\text{STD}} < m \wedge \text{CARD}(\mathbf{E}_e(\mathbf{ag})) = m]]$

Note that that condition $\text{CARD}(\mathbf{E}_e(\mathbf{ag})) = m$ in (18) will require that m -many beating events with distinct agents, and thus at least m agents where m is greater than the standard for MANY.

- A consequence of this semantic analysis is that third and fourth forms must be interpreted distributively, which is exactly what we want.

Recall the lack of such marking with collective predicates in (9-10).

The natural extension then to fourth forms like *ḏaḏbi:lj* ‘many burn many’, as seen in (19), is that we target both core argument thematic roles.

(19) *Ba:jach gwèjaláy nyuwí ḏaḏbi:ljkwi.*
 ba:-j-ch gwèjaláy nyu-wí ḏaḏbi:lj-k-wi
 person-PL-SUBJ trash 3/3.SUB.do burn.PL-SS-AUX/do

‘People are burning a lot of trash.’ (Watahomigie et al., 2001b, p. 247)

$$(20) \quad \text{đadbi:l}j \rightsquigarrow \lambda y \lambda x \lambda e [*BURN(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y \\ \wedge \exists m, n [MANY_{STD} < m, n \wedge CARD(\mathbf{E}_e(\mathbf{ag})) = m \wedge CARD(\mathbf{E}_e(\mathbf{th})) = n]]$$

There is evidence for treating the agent and theme cardinality conditions as two separate constraints. The reason is that they are separable.

- Watahomigie et al. 2001b notes that there are a small number of verbs that actually have fifth forms, which even further increases the object cardinality.
- The verb *sahák* ‘to hang’ is one such verb. Note that in addition to the fourth forms where ‘many hang many’, we have a fifth form that requires many hang even more things.

(21) Watahomigie et al. 2001b, p. 254

- sahák (wi) ‘to hang’
- sahájk ‘(a few/many) to hang one thing’
- đis’hák ‘(one) to hang many things’
- đis’hájk ‘(many) to hang many things’
- điđs’hájk ‘(many) to hang a lot of things’

These forms argue for treating the two cardinality constraints separately, as in (22), but also reinforces again the idea that verbal plurality in Hualapai involves gradable cardinality, ranging from few, to many, to a lot.

$$(22) \quad \text{điđs’hájk} \rightsquigarrow \lambda y \lambda x \lambda e [*BURN(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y \\ \wedge \exists m, n [MANY_{STD} < m \wedge ALOT_{STD} < n \\ \wedge CARD(\mathbf{E}_e(\mathbf{ag})) = m \wedge CARD(\mathbf{E}_e(\mathbf{th})) = n]]$$

With this analysis we have a comprehensive treatment of the formal semantics of the Hualapai plural verb paradigm, but we still do not have an account of the semantic scale.

- In particular, we cannot reduce the notion of the semantic scale to that of entailment.
- It’s just not the case that a sentences built on third form verbs with a high cardinality object entail minimal pairs with a second form verb, which merely requires a paucal subject.
- Thus, having a proposal for the truth conditions of such verbs cannot, on its own, give us an account of the semantic scale.

That said, this account does allow us to define a plurality scale based on properties of the semantic objects that satisfy the verbs in the paradigm as defined.

(23) **Semantic Scale** $<_s$ (**Informal**): Given V and V' of type $\langle e, \langle \epsilon, t \rangle \rangle$ —think subject and event arguments, the two arguments Hualapai verb paradigms concern— $V <_s V'$ just in case the smallest cardinality i+e for an individual/event-pair satisfying V has a lower cardinality than the smallest among such pairs satisfying V'

Let’s consider how this works.

- First form verbs in Hualapai can have a singular subject participating in a non-pluractional event. Thus, the smallest individual/event pairs have cardinality 2.
- In the second form paucal, the smallest individual/event pairs must have a cardinality greater than 2 because the subject alone, before even considering the event, must have a cardinality of at least 2.
 - The smallest such pairs would have cardinality 3 involving a dual subject participating in an atomic event.
- Third forms involve pluractionality. Such forms always involve an event argument with cardinality greater than $MANY_{STD}$, which must be greater than 3.
 - Thus, the smallest individual/event pairs satisfying second form verbs have a lower cardinality than those satisfying third form verbs.
- In fourth forms both the subject and event arguments must have a cardinality exceeding $MANY_{STD}$.
 - This must result in a larger cardinality than the smallest such pairs satisfying third form verbs, which only require one argument exceed $MANY_{STD}$.
- Finally, fifth forms involve standards even greater than many, i.e., $ALOT_{STD}$.
 - Critically, these forms, in every paradigm given by (Watahomigie et al., 2001b), have another argument that must exceed $MANY_{STD}$.
 - Clearly then smallest pairs satisfying $MANY_{STD}$ will be smaller than the smallest pairs satisfying $MANY_{STD}$ and $ALOT_{STD}$, respectively, given that $MANY_{STD} < ALOT_{STD}$.
 - Thus, fifth forms also behave according to the semantic scale in (23).

We now have an analysis, both of the truth conditions of the Hualapai verb forms, but also an analysis of the semantic order $<_s$.

- Forms are ranked higher, or we might say, “more plural”, if they involve more agents and more events.
- Critically, though, the distribution of those agents and events is such that forms do not entail each other.
- We thus have need of a semantic scale which exists alongside the truth-conditional content of the the various forms.

In the next section we turn to the morphological scale and show that, just as with the meaning of the forms, we can order the forms themselves along a scale of morphological complexity such that all things being equal, higher ranked forms should having meanings that are higher on the semantic scale.

3.2 Hualapai verbal number and the morphological scale

Recall that while we couldn’t uniformly treat the Hualapai verbal exponents in terms of accumulation, there were subcases which had this structure.

Form 1	Form 2	Form 3	Form 4	
gilgyo	gilgyo-j	gilgyo:	gilgyo:-j	‘tie’
hwal	hwa:l		hwa:l-j	‘dig’

In particular, doing $\alpha \oplus \beta$ must yield a more plural meaning than either doing α or β on its own.

- We use \oplus in this example because the main result of Baerman 2019 is that the Hualapai morphological order should have the structure of addition, even if it does not simply involve adding more exponents.

—We skip to example (30), but please read later—

- That is, it should form a commutative monoid with its standard algebraic pre-ordering.³
- When we try to make good on Baerman’s intent what we will see is that his account as presented fails to order all exponents correctly under the preorder induced by addition.
 - But, when make adjustments to ensure the ordering, we also resolve a morphological puzzle in Hualapai involving string equivalent forms with a reduplicated numeral prefix versus those with a sequence of numeral prefix and homophonous causative.

- We take this to be strong argument for Baerman’s account, which captures the algebraic structure of Hualapai number agreement, but also clarifies not immediately related morphological facts.

A commutative monoid is a set P closed under a binary operation that has a identity element and satisfies associativity and commutativity, which induces an algebraic preordering on P such that $x \leq y$ iff $\exists z[y = x + z]$.

- This is the familiar structure of the positive integers under addition, and we have already seen examples from Hualapai which suggest a similar structure
- Consider the paradigm for ‘tie something large’ from Figure 3.2, specifically $gilgyo-j \leq gilgyo:-j$.
 - We can say that forms with both length and the $-j$ suffix are greater than those with just length because there is something we could add to length, namely the $-j$ suffix, which would equal that higher form.
 - The same reasoning works to show that forms with the $-j$ suffix rank below those with both $-j$ and length.
 - This is perfectly parallel to the fact that $2 \leq 3$ and $1 \leq 3$ because there are integers, 1 and 2 respectively, such that $1 + 2 \leq 3$.

The question now is can we find this structure throughout the verbal morphology.

- To begin, Baerman provides the following hierarchy based on the empirical fact that should some meaning be assigned one of these exponents, those meaning higher on the semantic scale must not use exponents to the left of where we started.

$$(24) \text{ suff} \leq \text{length} \leq \text{suff} \oplus \text{length} \leq \text{prefix} \leq \text{prefix} \oplus \text{suff} \leq \text{prefix} \oplus \text{length} \leq \text{prefix} \oplus \text{length} \oplus \text{suff} \leq \text{prefix} \oplus \text{prefix} \oplus \text{suff}$$

Baerman tries to capture the additive structure of this morphological hierarchy by saying that these exponents expone numerical features. Assume the following correspondences.

$$(25) \begin{array}{l} \text{a. } \text{suff} \leftrightarrow 1 \\ \text{b. } \text{length} \leftrightarrow 2 \\ \text{c. } \text{prefix} \leftrightarrow 4 \end{array}$$

Then, the hierachy in (24) looks like (26).

³Baerman 2019 does not use these terms, but it is clearly the intent.

$$(26) \quad 1 \leq 2 \leq 1 \oplus 2 \leq 4 \leq 4 \oplus 1 \leq 4 \oplus 2 \leq 4 \oplus 2 \oplus 1 \leq 4 \oplus 4 \oplus 1$$

This system captures structural properties of the system familiar from addition.

- For instance, we must assert length is greater than suffixation, but with this fact establish we immediately capture the fact that prefixation along with suffixation must be less than prefixation with length.
- It follows from the fact that $a \leq b$ ensures $a + x \leq b + x$.

There are critical features, though, that this system lacks, which is present in commutative monoids.

- First, we require addition to be a total function. Here, though, note that there combinations not attested.
 - While we can have double prefixation with suffixation, there is no double prefixation with length. That is, $prefix \oplus prefix \oplus length$ is undefined.
- This tells us that we are actually dealing with a partial commutative monoid, defined in (27).
- More importantly, the fact that we have missing values in the system means that the ordering relation as given does not hold.

(27) A *partial commutative monoid* is a structure $(P, \oplus, 0)$, where P is a set, $0 \in P$, and \oplus is a partial binary operation on P satisfying the following properties, for all $x, y, z \in P$:

- *Associativity*: $x \oplus (y \oplus z)$ is defined if and only if $(x \oplus y) \oplus z$ is defined, and then the two values are equal.
- *Commutativity*: $x \oplus y$ is defined if and only if $y \oplus x$ is defined, and then the two values are equal.
- *Zero element*: $x \oplus 0$ is defined with value x .

The algebraic preordering on P is defined by

$$x \preceq y \quad \text{if} \quad \exists z (y = x \oplus z), \quad \text{for all } x, y \in P.$$

Note that algebraic preorder requires that a is less than b just in case we can find the difference between them, i.e., an object c such that $a + c = b$.

- The justifying relationship holds for all pairs in the feature preorder in (26) except for one, namely $1 \leq 4 \oplus 4 \oplus 1$.
- The issue is that we do not have number 8. There is no x feature hierarchy such that $x \oplus 1 = 4 \oplus 4 \oplus 1$.
- This should give us pause.

Baerman shows that Hualapai plural verb morphology could almost have structure of a partial commutative monoid, but there is something funny going on with $prefix \oplus prefix \oplus suffix$ forms.

- Actually, when we look at these forms there are other issues.
- Note, for instance, we also have a failure of associativity.
 - We don't precisely know the bracketing, but $(prefix \oplus (prefix \oplus suffix))$ should be defined if and only if $((prefix \oplus prefix) \oplus suffix)$.
 - This holds for our other triple-feature forms, namely $prefix \oplus length \oplus suffix$, where both $prefix \oplus length$ and $length \oplus suffix$ are both defined.
 - In contrast, $prefix \oplus prefix$ is not defined, and so $((prefix \oplus prefix) \oplus suffix)$ must be undefined, which by associativity requires that $(prefix \oplus (prefix \oplus suffix))$ be undefined apparently counter to fact.

This is all to say that if we want to treat Hualapai plural verb morphology as having an additive algebraic structure, which we can almost do, there is a problem to resolve with the double prefix forms.

- Interesting, there are independent reasons to think that these forms require an alternative analysis.
- As Baerman 2019 notes, there has been a question in the literature about whether to analyze certain cases of double prefixations as exactly that or as reduplication.
- The issue is that, as discussed in Watahomigie et al. 2001b, the most common causative forms, when they take a plural prefix, the prefix is a copy of the form of the causative.

We get paradigms that look like the following, where I have highlighted the doubled prefix.

- (28) a. jithul ~ **jj**thul:l ~ **jj**thul:l ~ **jj**thul:l 'wash'
 b. diboq ~ **di**diboqj ~ **di**dibo:q ~ **di**dibo:qj 'spill'

In earlier work, Redden 1966 and Watahomigie et al. 1982 treat these as reduplication, but in Watahomigie et al. 2001b, the authors of the latter work have revised their view in favor of the idea that we have homophony of two prefixes in this case.

- That is, in *jithul* we have a causative prefix *ji-*, which in the plural is preceded numeral prefix which happens to have the same form.

We bring up these forms because we can contrast them another class of stems which we believe do involve reduplication. These are verbs that begin with an *s-/th-* causative.

- (29) a. $sqwa:n \sim sqwa:nj \sim \mathbf{dis}qwa:nj \sim \mathbf{dids}qwa:nj$ ‘peel’
 b. $thigóm \sim thigómj \sim \mathbf{dith}gómj \sim \mathbf{did}thgómj$ ‘break’

Baerman 2019 analyzes these as involving a causative *di-* prefix, which only appears in higher ranking forms for these verbs, and which can then take a homophonous plural *di-* in the highest ranking forms.

- This allows him to maintain a formal similarity with the causatives in (28).

This analysis has a major drawback, though, which is that we do not lose the causative *s-/th-* in these higher number forms, which makes us skeptical that *di-* is a causative in these cases.

- We want to propose a counteranalysis where all the stems in (29) and (28) have a uniform causative prefix across all forms, whether *ji*, *di-*, *s-*, or *th-*.
- For the *ji* and *di-* casusatives, there is matching plural prefix.
- Critically, *s-* and *th-* causatives take the *di-* plural prefix (as Watahomigie et al. 1982, p. 254 says), which we see in forms like $\mathbf{dis}qwa:nj$.

When we move to forms like $\mathbf{dids}qwa:nj$, in virtue of treating *s-* as the causative and *di-* as a plural prefix, we have the option of treating the second *di-* as bona fide reduplication rather than a second plural prefix.

- This allows us to preserve that analysis in Watahomigie et al. 2001b for the *ji* and *di-* casusatives where we do not have reduplication, merely the addition of a homophonous plural prefix.
- This we distinguish from true reduplication of plural prefixes for high number forms of *s-*, *th-* and other causatives.⁴

We now have a solution to the problem of the morphological order.

- If the relevant double-prefixed forms are not two sequences of prefixes, but reduplication, a different exponent, then we can treat it as exponenting a different feature as in (30).

⁴It is perhaps not surprising that we do not see reduplication as an option in the *ji/di-* causatives. The reason is that for these stems it would involve a tripling of identical elements (the causative, it’s matching plural marker, and the reduplicant), which is perhaps a bridge too far.

—We pick things up here, bypassing too much morphology—

- (30) a. $\text{suff} \leftrightarrow 1$
 b. $\text{length} \leftrightarrow 2$
 c. $\text{prefix} \leftrightarrow 4$
 d. $\text{reduplication} \leftrightarrow 7$

We can combine these features in the attested ways which now correctly gives us a partial commutative monoid.

- Every relation in the order is supported by the addition operation and we no longer have the associativity problem.
- The sequence of two prefixes, which was undefined outside of the context of a suffix, is no longer treated as a sum.

(31) $\text{suff} \leq \text{len} \leq \text{suff} \oplus \text{len} \leq \text{pref} \leq \text{pref} \oplus \text{suff} \leq \text{pref} \oplus \text{len} \leq \text{pref} \oplus \text{len} \oplus \text{suff} \leq \text{redup} \oplus \text{suff}$

(32) $1 \leq 2 \leq 1 \oplus 2 \leq 4 \leq 4 \oplus 1 \leq 4 \oplus 2 \leq 4 \oplus 2 \oplus 1 \leq 7 \oplus 1$

Though we depart from Baerman’s analysis of the double-prefix forms, our reanalysis is a vindication of his approach.

- In particular, we take it to be a strong argument for the algebraic analysis.
- Assuming it led us precisely to an area of morphological analysis that was independently contentious (reduplication vs. double prefixation), and then pointed us to the correct analysis.
- With it, we now have a well-define morphological order in terms of a partial commutative monoid which stands alongside the semantic order we developed in the previous section based on the sum cardinality of verb stem arguments.

In the next section we bring semantics and morphology together.

3.3 Hualapai scale alignment via degree semantics

We are now in a position to think about compositional morphosemantics of Hualapai plural verb morphology.

- We will work in a roughly Distributed Morphology framework Halle and Marantz (1993), acknowledging that there are likely theoretical issues the Hualapai facts raise for the framework that we are not fully exploring.

The intuition behind the analysis the following:

- Recall how pronouns in English are often analyzed as denoting a variable x paired with a presupposition inherited from the gender features on that pronoun (e.g., Sudo 2012).

- (33) a. $pro_{[+nom,+fem]} \leftrightarrow \text{she} \leftrightarrow \text{FEMALE}(x) : x$
 b. $pro_{[+nom,+masc]} \leftrightarrow \text{he} \leftrightarrow \text{MALE}(x) : x$

- The exponents we see in Hualapai correspond to numerical features, which I propose are interpreted as presuppositions on the vP denotations, just like the gender features on pronouns.
- The presuppositions introduced by these numeral features will involve ordered cardinality standards—e.g., std_1 , std_2 , std_3 , etc.—which we can read of the numerical feature in question.
 - Note, I am not assuming that, for instance, the std_2 equals cardinality 2. These behave like other standards, in that the precise degree can vary, but the order between them must be respected.
- A particular form, corresponding particular features, will introduce a specific presupposition that will be compatible with various meanings for the vP in which it occurs—this is good, remember we don't have one-to-one mapping!
- But, the presuppositions in question will constrain the meanings of forms across a paradigm such that they are well-formed, i.e., they track the semantic order.

To begin, Hualapai plural verb morphology concerns both the event and external argument.

- For this reason we take it implicate the v° head that introduces introduces the external argument (and in event-semantic frameworks, relates it to the event argument).

- In particular, we take the v° head to host the ordered numerical features introduced in the previous section.



We know how these features will be dealt with at the morphophonological interface—due to uniqueness of featural decomposition in algebraic structure, we will insert a prefix and a suffix.

- The question is how to get the correct semantic interpretation.
- Critically, we want to understand how the semantic scale, repeated below from (23), is respected.

- (35) **Semantic Scale** $<_s$ (**Informal**): Given V and V' of type $\langle e, \langle \epsilon, t \rangle \rangle$, $V <_s V'$ just in case the smallest cardinality $i+e$ for an individual/event-pair satisfying V has a lower cardinality than the smallest among such pairs satisfying V'

We propose that two things happen at the interpretation of this v° head.

- First, we insert some truth conditional meaning \mathbf{M} in the context of the root in question—recall we have six possibilities:
 - Neutral
 $\mathbf{M}_1 = \lambda V \lambda x \lambda e [*V(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = x]$
 - Paucal
 $\mathbf{M}_2 = \lambda V \lambda x \lambda e [*V(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y \wedge \exists n [1 < n < \text{FEW}_{\text{STD}} \wedge \text{CARD}(\mathbf{ag}(e)) = n]]]$
 - Many Objects
 $\mathbf{M}_{3.1} = \lambda V \lambda x \lambda e [*V(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y \wedge \exists m [\text{MANY}_{\text{STD}} < m \wedge \text{CARD}(\mathbf{E}_e(\mathbf{th})) = m]]]$
 - Many Subjects
 $\mathbf{M}_{3.2} = \lambda V \lambda x \lambda e [*V(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y \wedge \exists m [\text{MANY}_{\text{STD}} < m \wedge \text{CARD}(\mathbf{E}_e(\mathbf{ag})) = m]]]$
 - Many Subjects and Many Objects
 $\mathbf{M}_4 = \lambda V \lambda x \lambda e [*V(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y \wedge \exists m, n [\text{MANY}_{\text{STD}} < m, n \wedge \text{CARD}(\mathbf{E}_e(\mathbf{ag})) = m \wedge \text{CARD}(\mathbf{E}_e(\mathbf{th})) = n]]]$

- Many Subjects and Even More Objects
 $\mathbf{M}_5 = \lambda V \lambda x \lambda e [*V(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y$
 $\wedge \exists m, n [\text{MANY}_{\text{STD}} < m \wedge \text{ALOT}_{\text{STD}} < m$
 $\wedge \text{CARD}(\mathbf{E}_e(\mathbf{ag})) = m \wedge \text{CARD}(\mathbf{E}_e(\mathbf{th})) = n]$

- Second, we insert a presupposition directly read off of the feature in question. We think of this like the presuppositional features of pronouns which accompany their truth conditions—i.e., a variable.

We propose the presupposition for a feature N has the following form:

- (36) $\text{MIN-CARD}(V_{\langle e, \langle e, t \rangle \rangle}) \geq \text{std}_N$, where
- a. $\text{MIN-CARD}(\alpha_{\langle \beta_1, \dots, \langle \beta_n, t \rangle \rangle})$ is the smallest cardinality of any sequence b_1, \dots, b_n satisfying α .

Let's make this concrete by continuing the example above. The meaning of the *vP* containing *jithul* 'wash' and prefixation+suffixation—i.e., feature 6 must be:

- (37) $\lambda x \lambda e. \text{MIN-CARD}(\mathbf{M}'_7(\llbracket jithul \rrbracket)) \geq \text{std}_6 : \mathbf{M}'_7(\llbracket jithul \rrbracket)(x)(e)$

Here we are obscuring the truth-conditions we are assigning to the verb with the 6 feature (i.e., the \mathbf{M}'_7), but...

- ... according to the presupposition, whatever meaning we assign, the smallest pairs that satisfy the result must have a cardinality greater than std_6 .
- This is not so informative on it's own, but when we compare to other features, we see that we now have a degree-based account of incremental morphology.

Note that we could have used reduplication—i.e., feature 7—with *jithul*, in which case we would have:

- (38) $\lambda x \lambda e. \text{MIN-CARD}(\mathbf{M}'_7(\llbracket jithul \rrbracket)) \geq \text{std}_7 : \mathbf{M}'_7(\llbracket jithul \rrbracket)(x)(e)$

While we may not know what meanings to assign to \mathbf{M}'_7 and \mathbf{M}'_7 , the presupposition ensures that we must pick a meaning for \mathbf{M}'_7 whose smallest satisfying pairs is smaller than those of \mathbf{M}'_7 .

- That is, if we say \mathbf{M}'_7 is the paucal (F2), then \mathbf{M}'_7 must not be the neutral (F1).
- Or, if we say \mathbf{M}'_7 has many objects (F3), then \mathbf{M}'_7 must not be the paucal (F2).
- But this is exactly the core generalization we started with! Incremental plurality in Hualapai thus falls out from:

- an algebra of features which gives us an order of morphological complexity.
- a presupposition based on those features that arranges possible *vP* meanings on a semantic scale—here the MIN-CARD scale.

- The result is that all paradigms are well-formed exhibiting scale alignment—i.e., $\alpha \leq_m \beta$ iff $\llbracket \alpha \rrbracket \leq_s \llbracket \beta \rrbracket$

A nice prediction of this kind of account, which is borne out, is that we predict syncretisms up the scale of meaning.

- For instance, if we pick the paucal (F2) meaning for *jithul*+6, committing to the minimal elements satisfying it meaning because greater than std_6 , then we could also use that same form for the "many object" (F3) meaning because it would also satisfy that same presupposition.

- In fact, this is the case for *jithul*+6—i.e., *jijthu*:1, which is use for the paucal (F2) and the many object (F3) readings.
- It is very common in the Hualapai system to make just two distinctions, i.e., neutral vs paucal and all stronger meanings Watahomigie et al. 1982, pg. 223-274.

The presupposition-based scale-alignment system developed here predicts this!

4 Conclusions

What can we take away from this work so far, beyond an analysis of Hualapai verbal plural marking?

- First, that incremental systems exist and need a compositional semantic analysis. They are not simply iconic.
- But, given the lack of one-to-one mapping between forms and meanings, it becomes difficult to do composition by assigning each form a meaning.
- What we have been arguing for here in Hualapai (and what we would argue for in Seri) is that the exponents we see on the surface are not exactly telling us about the meanings of verbs.
- Instead, they reflect presuppositions on the range of possible meanings for those verbs.
 - In the case of Hualapai, choosing some exponent α over β is about signaling how big the arguments of whatever function we end up with tend to be.

- There is thus a kind of lossy relationship between the forms and the meanings. The forms we have, especially in relation to other forms, give us through their associated presuppositions a range of possible meanings they are compatible with.
 - And this range of meanings can have meaning-based structure to it (i.e., the semantic order).
- That this way of organizing your morphosemantics interface exists is the primary takeaway of this talk. I hope it has intrigued you.

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